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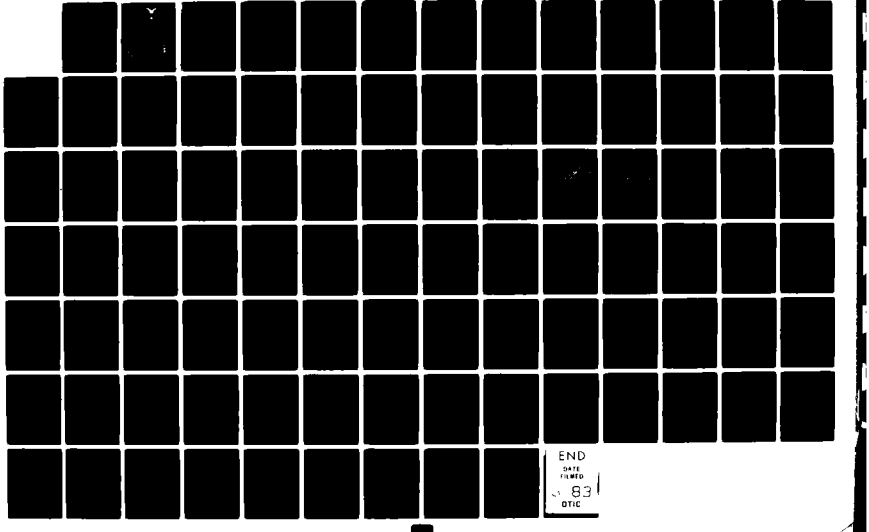

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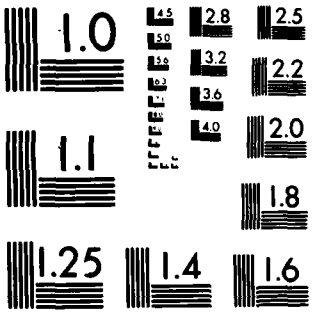

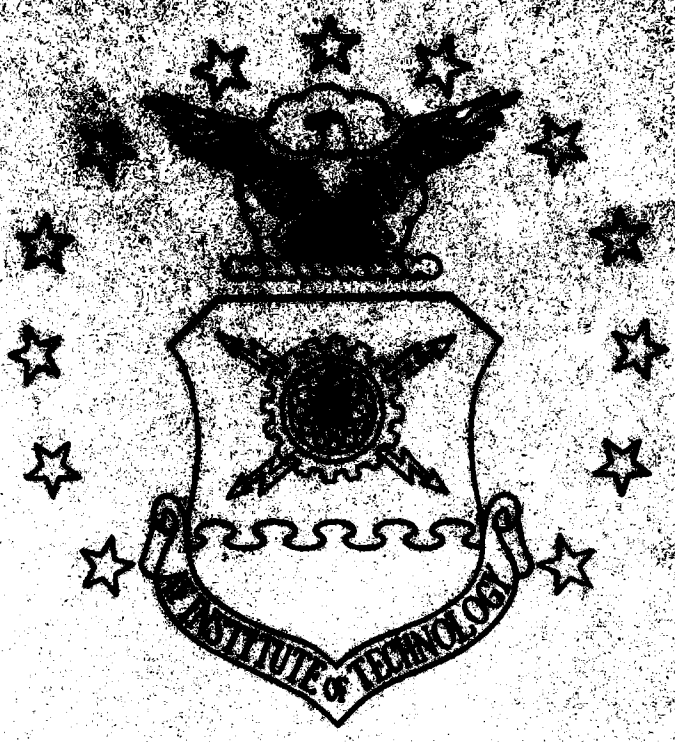

DEPARTMENT OF THE AIR FORCE

AIR UNIVERSITY (ATC)

AIR FORCE INSTITUTE OF TECHNOLOGY

Wright-Patterson Air Force Base, Ohio

AFIT/GSO/MA/82D-1

A MONTE CARLO TECHNIQUE SUITABLE FOR OBTAINING COMPLEX SPACE SYSTEM RELIABILITY CONFIDENCE LIMITS FROM COMPONENT TEST DATA WITH THREE UNKNOWN PARAMETERS

THESIS

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Major Canadian Forces

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THESIS

Presented to the Faculty of the School of Engineering

Of the Air Force Institute of Technology

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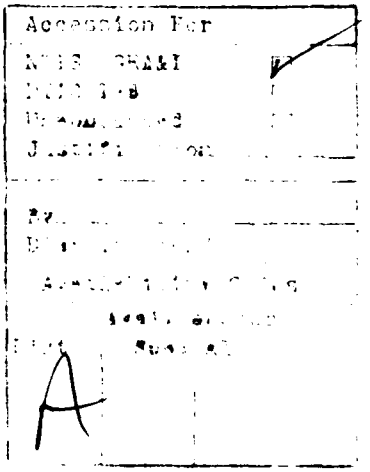
Requirements of the Degree of

Master of Science



by

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Graduate Space Operations

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Preface

A common problem in estimating spares requirements for complex systems is accurately forecasting failure rates. While a point estimate of system reliability can normally be obtained, it is relatively meaningless without some confidence limit on it. The problem is compounded in highly reliable systems that have some period of failure free life before entering their failure period. Considerable work has been done on reliability estimation over the last twenty years at the Air Force Institute of Technology. This thesis is a continuation of many efforts to provide a flexible, robust method of estimating confidence limits. In an attempt to make the methods and procedures developed as useful as possible to future readers, I have included several illustrations, an example of how to use the procedures, and the computer printouts of the programs used.

Thanks are due to Dr. Albert H. Moore for his assistance in selecting this topic and his guidance throughout. Also, the frequent support of the AFIT and ASD computer personnel in extending my account and providing timely advice on methods to improve turn-around time is much appreciated.

Murray R. MacDonald

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Abstract

A Double Monte Carlo method of obtaining confidence limits for complex systems based on component failure data assuming a three parameter Weibull distribution was developed. Three new parameter estimation routines were developed and compared with the Harter-Moore three parameter maximum likelihood routine for use with the Monte Carlo method. The sensitivity of the method to system reliability, sample size, and number of points in the component reliability distributions was assessed. An approximate method of calculating and correcting for parameter estimation bias was developed and illustrated. The Double Monte Carlo method appears to be effective at system reliabilities from 74% to 96% with componenent failure sample sizes as small as five with the Linear Least Squares parameter estimation routine developed.

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I Introduction

Problem Statement

Accurate estimation of complex system reliability is important for operational planning and system replacement scheduling. Estimation of system reliability for Space systems is particularly important in that replacements or repairs are generally difficult to make. Little system reliability data exists due to the small numbers of each type of system and the varying complexity of the systems. Component reliability continues to improve and longer missions are planned and conducted (Ref 32) so empirical estimation of system reliability is impractical, thus necessitating use of an analytic model to estimate system reliability. Most models simply assume a constant failure rate (exponential distribution) for the components (Ref 19), but these models have proved to be very conservative which results in larger system purchases than required (Ref 2). Empirical data was added to the models in an effort to improve their accuracy and the effect of mission controller selection of alternate system modes (work-arounds) has also been considered (Ref 25) to produce reliability estimates which are not grossly in error.

Space system failures have tended to occur in one of two separate periods: early system failure caused by undetected defects, and wearout or failures caused by random mishaps later in the system life (Ref 2:6). Accurately predicting early failures caused by defects is a function of

quality control. With improving design and quality control, complex spacecraft with failure modes dominated by wearout can be manufactured (Ref 2:7), and the effect of a few defects can be nullified by work-arounds. The estimate of increasing importance is the effect of long term failures. To accurately model these effects, the system component failure distributions must include location parameters.

Incorporating location parameters into models which use an exponential distribution assumption would account for the necessary period before the long term failure period was entered. Unfortunately, the exponential distribution is not robust in that departures from the distribution can result in large errors (Ref 11). The primary use of the exponential distribution in system reliability estimates would appear to be for a system which is composed of many components which are changed on failure (Ref 14:235,237). This is not the case for space systems, so an alternate set of assumptions which can more accurately model the failure distributions encountered must be used.

A point estimate of system reliability is relatively easy to obtain but is of itself of little value without confidence bounds. For example, a system reliability estimate of 0.99 with a 90% lower bound of 0.95 is considerably different than a system reliability estimate of 0.99 with a 90% lower bound of 0.50. The system reliability estimates required are a point estimate and a lower limit at some pre-selected confidence level.

The problem is to develop a robust model that can account for some period of guaranteed life before entering the wearout period for reliability confidence interval estimates of complex systems. The model must be able to incorporate many different types of components with different failure modes and guaranteed lives.

Review of Applicable Literature

Over the last twenty years, a considerable amount of research has been done into estimating component parameters from failure data and system reliability from component data. Orkand in 1960 (Ref 24:18) suggested a Monte Carlo method for the estimation of the lower confidence limit on the reliability of a complex network of components. He suggested this method as a general solution procedure and provided a more detailed solution for the case where the sample data for each component are in binomial form. Rosenblatt in 1962 discussed the problem of determining confidence limits for the reliability of complex systems. She suggested that estimation using simulation was ". . . the simplest and most generally applicable procedure for estimating R through F $(x_1,\ldots,X_k)\ldots$" (Ref 29;119) and used a binomial theoretical treatment of the problem.

In 1963, Quayle (Ref 27) summarized the applicable reliability theory and provided some preliminary work on parameter estimation with his method of using order statistics to estimate the scale parameter of the Weibull probability density function. The same year Bernhoff (Ref 3) showed that adding component confidence limits to obtain system confidence limits was erroneous and that no single system parameter was appropriate when the components have different distributions. He determined that "The analytical solution becomes impractical when the system reliability estimator is the function of two or more dissimilar mathematical forms and mathematical simulation must be used" (Ref 3:3). For confidence limits he generated and used a step cumulative distribution function.

Levy in 1964 (Ref 16) used a Monte Carlo Technique to obtain system

reliability confidence limits from component failure test data assuming that the components' failure distributions were two-parameter Weibull (location parameter = 0). He established a step cumulative distribution function to obtain system lower reliability confidence limits. His work was later consolidated and published (Ref 17). Moore in 1965 (Ref 23) extended the concept of using the Monte Carlo technique for obtaining system reliability confidence limits from component data for cases where the mathematical model for the underlying failure distributions is known, component test data exists to estimate the parameters, and the distribution of the estimators of the parameters is unknown. The basic method consisted of obtaining a sample distribution of reliabilities from which an approximate confidence interval, or limit, can be obtained at any level of confidence.

In 1967 Hahn and Shapiro in their text (Ref 10:Chap 7) discussed the problem of estimating confidence intervals for complex systems. The methods developed were the use of the Central Limit Theorem for series systems with a large number of components, the generation of system moments, and the Monte Carlo method. They favor the generation of system moments for relatively simple systems but prefer the Monte Carlo method for "... highly complex situations for which the method of generation of system moments becomes too difficult." (Ref 8:246)

In 1972 Lannon (Ref 15) used the Monte Carlo method to approximate system reliability confidence limits assuming the components had failures characterized by two-parameter Weibull distributions (location parameter=0). In 1973 Boardman and Kendall (Ref 6) developed a method of parameter estimation for a binomial mixture of two single parameter exponential distributions under the assumption of two possible causes of

failure, each with a single parameter exponential distribution. Their method may have some application to the problem of estimating spacecraft reliability but suffers from the assumption of location parameters=0, the lack of robustness of the exponential distribution, and the simplicity of the model.

In 1976 Gatliffe (Ref 9) extended the use of the Log-gamma procedure for estimating system reliability from series only arrangements to series-parallel arrangements. His method does not require either assumptions about failure distribution of any component, or equal sample sizes. His results are good for highly reliable systems but the accuracy is unstable from configuration to configuration. His system also can be very conservative in that it generates artificial failures when no failures were observed.

Bilikam and Moore provided two practical illustrations of the use of the Monte Carlo method to estimate mission reliability in 1977 and 1978. In the first (Ref 4) they used time-grouped mission equipment failure data where the exact failure times were unknown although the equipment was known to have failed during a mission of known length. In the second (Ref 5) they used known failure times of one type of aircraft engine component. Also in 1978, Snead (Ref 30) developed a univariate method of using the asymptotically normal property of $\hat{R}(t)$ with a Monte Carlo technique to estimate system reliability confidence limits.

In 1979 Putz (Ref 26) used the univariate Monte Carlo method to estimate lower confidence limits of system reliability based on component test data. He assessed the sensitivity of the method to the asymtotic normality assumptions and estimated the minimum sample size required for this method. His method is the most effective when component and system

reliabilities are low (less than 0.9) and sample sizes of fifty or more are available. When the component reliabilties are high and/or the sample size is low, the distribution of R(t) is no longer nearly normal which can result in significant errors.

In 1979 Rice (Ref 28) assumed the number of component failures was binomially distributed. Using the asymtotic normality property of the binomial distribution (n>20), he developed a Monte Carlo method of estimating lower confidence limits on system reliability with component failure data input. In the cases where no failures were observed he used the Gatliffe method of generating artificial failures. Also in 1979, Antoon (Ref 1) used Monte Carlo analysis to find empirically the standard deviation of reliability of a system whose underlying component distributions were two-parameter Weibull (location parameter=0). He developed, by curve fitting, an equation for computing the standard deviation in terms of reliability and sample size.

In 1980, Johnston (Ref 13) used a Modified Double Monte Carlo procedure to estimate system reliability from component data where the component failure distributions were characterized by the two-parameter Weibull (location parameter=0). He used the bias tables published by Thoman, Bain and Antle (Ref 31) to correct his estimates of reliability and obtained reasonable results, although the results are difficult to evaluate fully since different system configurations with different reliabilities were used. Also in 1980 Moore, Harter, and Snead (Ref 21) compared three Monte Carlo techniques for obtaining system reliability confidence limits; the bivariate, the univariate, and the Double Monte Carlo. Their conclusion was that the bivariate tended to be conservative and the univariate asymtotic optimistic with the Double Monte Carlo in

between. Depuy (Ref 7) compared the accuracy of two Monte Carlo simulation techniques of finding lower system reliability confidence limits: the bivariate and the univariate. She found the bivariate method the most accurate if the true system reliability is below 0.95, and the univariate most accurate if the true system reliability is greater than about 0.95 and the component data sample size is less than twenty.

Finally, in his review of reliability growth (Ref 33), Vonloh discusses the use of the Monte Carlo method in system reliability growth prediction models. The entire subject area of reliability growth is applicable to new developing technology and the flexibility of th Monte Carlo method in general makes it useful for estimation of the reliability of systems whose parameters may be changing.

Model Selection

Two methods of system reliability determination warrant further discussion: the method of moments and the Monte Carlo method. The method of moments can be the most economical aproach. It also allows the analysis of the relative importance of each component variable by examination and does not require any assumptions about the underlying component distributions. On the other hand, the accuracy of the results is not always consistent and cannot be readily analyzed. Also, the generation of system moments soon becomes unworkable with increasing system complexity (Ref 10:246,247).

The Monte Carlo method requires that an assumption regarding the component failure distribution be made. Also, it does not allow for detection of dominant components. Since the method estimates overall

system performance, a change in any of the components requires that the entire system be re-analyzed. The method also requires a considerable amount of computer time - the exact amount being dependent on the system and the assumptions in the model. On the positive side, the Monte Carlo method has proven useful in a wide variety of applications. It has been extensively used in developing system reliability confidence limits, particularly for a two-parameter Weibull or a binomial distribution. It is easy to use and, if the Double Monte Carlo method is used (Ref 20) does not require any assumptions other than those of the component reliability distributions. Therefore, for this problem, the Double Monte Carlo method was selected as the most suitable for developing the required model.

Underlying Failure Distribution Selection

In using the Monte Carlo method, it is necessary to select a suitable component failure distribution. The distribution should allow for the possibility of a location parameter greater than zero and a non-symmetrical shape in order to allow fitting of the distribution to the data available or, as Easterling wrote "Thus the task facing the statistician is more often one of model fitting than of parameter estimation" (Ref 8). This is because a given set of data may not clearly resolve the appropriate distribution.

Exponential. The exponential distribution is widely used and is well analyzed. However, even if a location parameter were added it would still not have the required flexibility in shape.

Gamma. The gamma distribution has been used in fatigue and wearout studies. It can assume a variety of forms which could be fitted to a considerable variation on data.

Normal. The normal density also is commonly used. It can accomodate a period of near-guaranteed life and can assume different scales depending on the mean and variance. However, its symmetrical shape limits its application.

Log Normal. Like the normal density the log normal can accommodate a period of near-guaranteed life and can assume different scales. However, its shape is limited to a positively-skewed normal curve.

Weibull. The three-parameter Weibull can accomodate any positive location parameter and a wide range of shapes and scale depending on the respective parameters. The scale parameter determines the spread about the mean, the shape parameter determines the failure rate - whether increasing, decreasing, or constant, and the location parameter determines the point beyond which failure can occur. The Weibull density function has shapes that are similar to the Gamma or the lognormal - assuming appropriate Weibull parameters. If the shape parameter is 1, the Weibull becomes an exponential function; a shape parameter of about 3.7 yields an excellent approximation to the normal function and a shape parameter of 2 can approximate Beta distributions. It has also been shown valid for a wide variety of actual situations (Ref 34) and has the necessary flexibility to fit any foreseeable set of failure data. Therefore, the three-parameter Weibull distribution was selected for this model.

Objectives

The objectives of this thesis are:

1. to develop a model to estimate complex system lower reliabiiity confidence limits;

2. to estimate the minimum practical sample size, and

3. to assess the sensitivity of the Double Monte Carlo method to the number of points in the sample distributions of reliability.

Assumptions

It is assumed that:

1. The underlying component failure distributions can be modeled by three-parameter Weibull distributions;

2. Components fail independently; that is, there are no secondary failures;

3. A mathematical relationship between component reliabilities and system reliabilities can be established;

4. The International Mathematics and Statistics Library (IMSL) subroutines GGUBS and GGWIB provide valid random variables; and

5. The user has a basic knowledge of reliability theory, Monte Carlo methods, and FORTRAN 77.

Approach

Existing methods of estimating parameters from failure data were examined and the most suitable method was selected for inclusion in the model. The failure data was generated artificially to represent true component failures from three-parameter Weibull distributions with different parameters. A single complex model was developed and the true reliability calculated analytically to use as a test of the model results. The model was tested at system reliabilities of about 75%, 85%, and 95% with all of the component reliabilities roughly matched to simulate a balanced system. At each reliability level component failure sample sizes ranging from five to fifty were modeled to assess the effect

of different sample sizes on the model accuracy.

The Double Monte Carlo method was used to generate the estimates of system reliability confidence intervals. When initial results indicated that the method of parameter estimation used was inadequate for a three parameter model, three new methods were developed, evaluated, and the best selected for use. The evaluation of the overall method consists of comparing the percent of times the X percent confidence interval captures the true reliability. For example, at the 80% confidence level, 80% of the time the confidence level should be below the true reliability. Finally, a sample illustration of the method was provided for practical guidance.

II Supporting Methods Development

Method of Maximum Likelihood

The assumption of a component failure distribution necessary for the Monte Carlo method requires that the distribution parameters for each component be estimated from its failure data. The method of maximum likelihood has been widely accepted as one of the most reliable methods of estimating distribution parameters. The maximum likelihood estimators are consistent, asymptotically normal and asymptotically efficient for large samples under most conditions (Ref 35:89).

The probability density function of a random variable T having a Weibull distribution with location parameter c, scale parameter θ, and shape parameter k is

$$f(t;c,\theta,k) = \frac{k}{\theta}\left(\frac{t-c}{\theta}\right)^{k-1} \exp\left[-\left(\frac{t-c}{\theta}\right)^{k}\right] \qquad o \le c \le t,\ \theta > o,\ k > o$$

To establish the maximum likelihood values of the parameters c, θ, and k it is necessary to formulate the likelihood function and solve for the values of the parameters that maximize the function. Let T_1, T_2, ..., T_n be the observed values in a random sample of size n. Then the likelihood function is

$$L(t,c,\theta,k) = \prod_{i=1}^{n} f(t_i;\ c,\theta,k) \qquad t_i > c$$

Now if the t_i are treated as fixed constants, then the likelihood function may be treated as a function of the three unknown parameters. Substituting in the value of $f(t_i;\ c,\ \theta,\ k)$ the likelihood function becomes

$$L(c,\theta,k) = \prod_{i=1}^{n} \frac{k}{\theta}\left(\frac{t_i-c}{\theta}\right)^{k-1} \exp\left[-\left(\frac{t_i-c}{\theta}\right)^{k}\right] \qquad \begin{array}{l} o \le c \le t_i \\ \theta > o \\ k \ge o \end{array}$$

The natural logarithm of the likelihood function, ln L, is easier to work with and does not result in any loss of generality since the maximum

of ln L and the maximum of L will occur at the same values of c, e, and k. The first partial derivatives of ln L with respect to each of the variables (the three unknown parameters) are set equal to zero and solved simultaneously to yield the maximum likelihood values of the parameters. The analytic solution of this system of partial differential equations is intractable and so requires an iterative computer routine. A commonly used routine which has proven satisfactory for similar applications is the Harter-Moore method of false position (Ref 12). This routine estimates the maximum likelihood parameters based on the first m order statistics of a sample of size n with r censored from below. The formulation of the natural logarithm of the likelihood function used is

$$\ln(L) = \ln(n!) - \ln((n-m)!) - \ln(r!) + (m-r)(\ln(k)-k\ln(e)) + (k-1)\sum_{i=r+1}^{m}\ln(t_i-c) - \sum_{i=r+1}^{m}[(t_i-c)/e]^k - (n-m)[(t_m-c)/e]^k + r\ln\{1-\exp[-(t_{r+1}-c)^k/e^k]\}$$

This formulation leads to the partial differential equations

$$\frac{\partial \ln L}{e} = -\frac{k(m-r)}{e} + k\sum_{i=r+1}^{m}(t_i-c)^k e^{k+1} + \frac{k(n-m)(t_m-c)^k}{e^{k+1}} - kr(t_{r+1}-c)^k \times \exp[-(t_{r+1}-c)^k/e^k]/e^{k+1}\{1-\exp[-(t_{r+1}-c)^k/e^k]\}$$

$$\frac{\partial \ln L}{\partial k} = (m-r)(1/k-\ln e) + \sum_{i=r+1}^{m}\ln(t_i-c) - \sum_{i=r+1}^{m}[(x_i-c)/e]^k \ln[(x_i-c)/e] - (n-m)[(t_m-c)/e]^k \ln[(t_m-c)/e] + r(t_{r+1}-c)^k \ln[(t_{r+1}-c)/e] \exp\{-[(t_{r+1}-c)/e]^k\} + e^k\{1-\exp[-(t_{r+1}-c)^k/e^k]\}$$

$$\frac{\partial \ln L}{\partial c} = (1-k)\sum_{i=r+1}^{m}(t_i-c)^{-1} + ke^{-k}\sum_{i=r+1}^{m}(t_i-c)^{k-1} + (n-m)ke^{-k}(t_m-c)^{k-1} - kr(t_{r+1}-c)^{k-1} \times \exp[-(t_{r+1}-c)^k/e^k]/e^k\{1-\exp[-(t_{r+1}-c)^k/e^k]\}$$

The routine which solves these equations is listed in Appendix B.

Median Rank Values

The Double Monte Carlo method builds estimated reliability distribution functions from which the appropriate confidence limits are selected. This requires that the randomly generated reliabilities be ordered and ranked. Several methods of rank plotting are available, with the median rank method the most commonly used because of the assumption that the rank distributions are skewed. The median rank is actually an incomplete beta ratio which cannot be readily calculated. However, the approximation to the median rank value given by

$$\tilde{t} = \frac{j-0.3}{n+0.4}$$

has an insignificant error for the large sample sizes ($n > 50$) used in the reliability distribution functions. An illustration of the median rank plotting against reliability with linear interpolation between points as used in this development is provided in Fig 1.

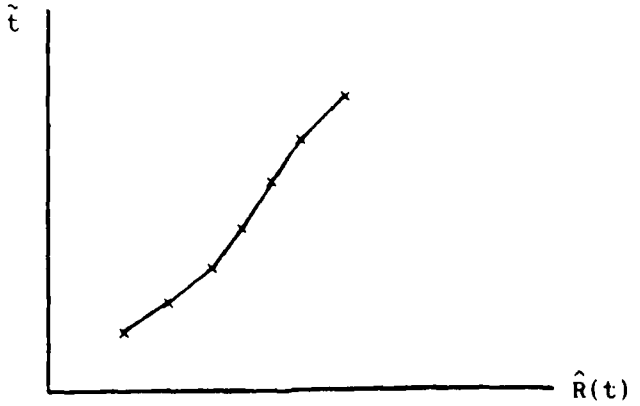

Figure 1. Median Ranks

Random Variable Generation

The IMSL routine GGWIB was used to generate single parameter (k)

Weibull variables which were transformed to three-parameter Weibull variables by subroutine WEIBL. The IMSL routine uses a c.d.f. $F(t)=1-\exp[-t^k]$ and inverts this to the reliability function to obtain the relationship $t=[-\ln(u)]^{1/k}$ where u is a uniform pseudo-random variable.

For the three-parameter distribution, the reliability function is

$$R(t) = \exp\left[-\left(\frac{t-c}{\theta}\right)^k\right]$$

Letting $u=R(t)$ and taking the logarithm of both sides gives

$$\ln u = -\left(\frac{t-c}{\theta}\right)^k$$

$$(-\ln u)^{1/k} = \frac{t-c}{\theta}$$

$$t = \theta(-\ln u)^{1/k} + c$$

Double Monte Carlo Method

The Double Monte Carlo method does not require any asymtotic assumptions and can be used with any component *failure data providing* that an assumption is made regarding the underlying failure distribution. For the purpose of developing and proving the model, the "real" failure data was generated using the "true" parameter values for each component. These true parameter values also allowed the analytical calculation of the true reliability which was used as a test of the results. The Double Monte Carlo method initially used consisted of the following steps.

1. Generate the true component failure data.
2. From the true component failure data, estimate the three parameters of each of the component reliability functions.
3. Generate a simulated sample of component failures, using the

estimated parameters, with the same number of observations as the test data.

4. From these simulated failures, estimate the three parameters of each of the component reliability functions.

5. Using the second estimate of the parameters, calculate the reliability of each component, $\hat{R}_{ij}$.

6. Repeat steps 3-5 until the required underlying sample size is obtained (50, 75 or 150).

7. Establish sample estimated reliability distribution functions for each component by ordering the $\hat{R}_{ij}$ for each component and matching each R_{ij} with the appropriate median rank. The first and last order statistic, associated with the median ranks 0 and 1 respectively, are approximated using linear extrapolation off the two nearest order statistics.

8. Randomly select a reliability for each component from its reliability distribution function using linear interpolation between points and compute the system estimated reliability $\hat{R}_{sl}$. Repeat until 600 estimates of system reliability are obtained.

9. Order the $\hat{R}_{sl}$ against median ranks and determine the 99, 95, 90, 80, 70, 60, and 50 percent lower confidence points using linear interpolation between points on the system sample distribution of reliability estimates. Note if the true reliability is greater than or equal to each of these confidence points.

10. Steps 1-9 provide one estimate of the system reliability confidence limits. To validate the method, these steps are repeated 1000 times. The X percent confidence limit should be less than or equal to the real system reliability X percent of the time.

Appendix A contains the computer program listing for the Double Monte Carlo method used. In actual practice only one true set of data would be available; step 9 would consist of printing out the desired confidence levels and step 10 would not be applicable.

III Method Development

System Configuration and Reliability

The Monte Carlo method of determining approximate system reliability confidence limits was selected as the most suitable for this development because of its advantages in handling complex systems. Of interest is the accuracy of the method for different reliability levels and different sample sizes for a complex system. Therefore, only one system configuration was used in order to be able to compare the results at the different reliability levels and sample failure levels tested. The system configuration selected is illustrated in Fig 2.

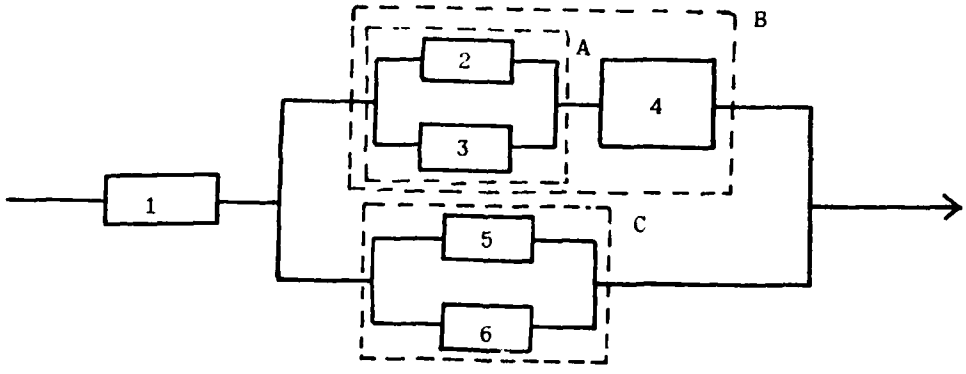

Fig 2. System Configuration

For the Weibull distribution, the component reliabilities are

$$R_i(t) = \exp\left[-\left(\frac{t-c_i}{\theta_i}\right)^{k_i}\right] \qquad t \geq o \quad c_i \leq t \quad \theta_i > o \quad k_i > o$$

From the system illustration, the system reliability R_s is

$$R_s = R_1 (1-F_B F_C) = R_1 (1-(1-R_B)(1-R_C)) = R_1 (R_B + R_C - R_B R_C)$$

$$R_B = R_A R_4$$

$$R_A = (1-F_2 F_3) = R_2 + R_3 - R_2 R_3$$

$$R_C = R_5 + R_6 - R_5R_6$$

$$R_s = R_1 \{[(R_2 + R_3 - R_2 R_3) R_4] + (R_5 + R_6 - R_5R_6) - [(R_2 + R_3 - R_2 R_3) R_4] (R_5 + R_6 - R_5R_6)\}$$

Parameters and Reliabilities Selected

As a matter of convenience and to maintain continuity with previous methods, a time of 100 units (T = 100) was used throughout. The same location parameters were used with the scale and shape parameters changed to provide balanced component reliabilities and a good range of parameter selections for the test. The parameters and reliabilities used are listed in Table I.

Estimates Generated

The parameter estimates generated by the method of maximum likelihood are biased, and the reliability estimates derived from these estimated parameters will be biased. Thoman, Bain, and Antle (Ref 31) empirically determined and tabled the bias in $\hat{R}(t)$ for a two-parameter Weibull distribution for a range of $.50 \leq \hat{R}(t) \leq .98$ and sample sizes from 8 to 100. For sample sizes greater than 15, the biases were only third decimal place values. Moore, Harter, and Antoon (Ref 22) assess the reliability estimate for a two-parameter Weibull as being very nearly normal and very nearly unbiased for sample sizes greater than about 20. The bias in $\hat{R}(t)$ for a three-parameter Weibull distribution has not been tabulated but, from the work done on the two-parameter Weibull distribution, can be expected to be small for larger sample sizes (> 30). The results of this method include any bias present, and should provide some feel for the magnitude of the bias in the parameter estimation routine used.

TABLE I

Parameters and Reliabilities

i =	1	2	3	4	5	6
c_i (Location) k_i (Shape) θ_i (Scale) R_i R_s = 0.741	10 2.8 140 .748	0 1.1 400 .804	15 2.0 180 .800	30 2.5 120 .771	25 1.7 180 .798	50 1.4 150 .807
c_i k_i θ_i R_i R_s = 0.849	10 2.3 200 .853	0 0.8 470 .748	15 2.0 180 .800	30 3.5 120 .859	25 1.2 250 .790	50 2.0 150 .895
c_i k_i θ_i R_i R_s = 0.959	10 2.9 270 .960	0 2.1 470 .962	15 2.3 400 .972	30 3.0 160 .920	25 2.7 250 .962	50 2.0 280 .969

Accuracy of Method

With Monte Carlo simulation, the larger the number of trials in the simulation, the more precise the solution will be. The desired degree of precision can be obtained by increasing the number of trials. The number of trials required for a degree of precision, E, can be calculated at a desired confidence level, $1 - \alpha$, by considering the Monte Carlo as a binomial problem where the estimate of interest is the proportion p of systems above a certain level. The calculation of n depends on the value of p actually found in the Monte Carlo simulation so a certain amount of trial and error is required. However, for a conservative estimate the largest n will be required for p = 0.5 which may be used to obtain an

upper bound on n. The normal approximation to the binomial can be used for convenience with little error since the number of Monte Carlo trials will generally be sufficiently large to ensure that np or n (1-p) are greater than five. This approximation leads to the relation

$$n = \frac{p(1-p)}{E^2} \; Z^2_{1-\alpha/2}$$

for a two sided interval where $Z_{1-\alpha/2}$ is the $(1-\alpha/2)100$ percent point of the standard normal distribution.

If the error is required, it can be determined by the relation

$$E = \left(\frac{p(1-p)}{n}\right)^{\frac{1}{2}} Z_{1-\alpha/2}$$

For example, if 1000 Monte Carlo trials result in 900 points within some specified tolerance, $p = \frac{900}{1000} = 0.9$ and, at the 95% confidence level

$$E = \left(\frac{(.9)(.1)}{1000}\right)^{\frac{1}{2}} \; 1.96 = \pm \; 0.0186$$

While the accuracy of the Monte Carlo procedure can be readily estimated, the model accuracy is more dependent on the accuracy with which the component parameters can be estimated from failure data. An error in parameter estimation is compounded by the use of these estimated parameters to generate random samples from which the second estimate of parameters is made. Therefore, the overall accuracy of the method can only be estimated from the results tested against the known point. An underlying assumption in developing the model is that the results can be extended to other similar complex systems.

IV Preliminary Results and Method Development

Preliminary Results

The Double Monte Carlo program was developed and run for a single estimate of system reliability (in step 10 only one estimate obtained) at $R_s = .95$ with 10 failures and 75 points in the sample distributions of component reliability estimates. This single estimate required 677 seconds of CDC Cyber Model 74 (CSB System) CPU time. When extrapolated to 1000 estimates per run and 15 runs required (5 sample sizes and 3 reliabilities), this results in an estimate of over 2820 hours (4 months) of CPU time. The expensive part of the method was step Four: calculating the second maximum likelihood estimators of the parameters from the simulated failures. In addition to being time consuming, this step produced estimates of the location parameter, c, that were larger than the test time (T = 100) for 202 of the 450 (45%) parameter estimations from the simulated failure data. These large estimates of c were not surprising in light of the high component reliabilities and the small sample size but did indicate a potential problem with the parameter estimation method under these circumstances.

Parameter Estimation Development

These initial results showed that a much faster and more reliable method of parameter estimation had to be obtained in order to reduce the CPU time and get more reliable estimates of the location parameter c. Because of the large number of parameter estimations required, the overriding requirement was to greatly increase speed. Other work done on parameter estimation was more concerned with accuracy (Ref 18), so the other available routines were also slow. The first approach taken was to

modify the maximum likelihood routine used. Initially the accuracy tolerances were set at .0001 and the program could run for a maximum of 550 iterations. Trial and error with the method resulted in reducing the maximum number of iterations to 300 and the accuracy tolerances to .01 without any significant degradation of results. This reduced the run time to about 60% of its previous level. Since this was still far too slow, modifying this procedure was abandoned and three new methods were developed.

The first approach taken was to develop a computerized graphical estimation technique using the ordered samples t_i, i = 1, 2, ..., n, and accept the parameters which gave the minimum error least squares fit.

The cumulative distribution function for the two-parameter Weibull, $F(t) = 1-\exp[-(\frac{t}{\theta})^k]$ can be rearranged and the natural logarithm taken twice to give the relationship

$$\ln(\ln(\frac{1}{1-F(t)})) = k\ln(t) - k\ln(\theta)$$

which, when rearranged, gives

$$\ln(t) = \frac{1}{k}\ln(\ln(\frac{1}{1-F(t)})) + \ln(\theta)$$

The substitution of $Y = \ln(t)$, $m=\frac{1}{k}$, $X=\ln(\ln(\frac{1}{1-F(t)}))$, and $a = \ln(\theta)$ provides the linear relationship $Y = mX + a$. The values of F(t) were estimated by the use of median ranks. The value of k was estimated from the first and last values on the abscissa and ordinate. Using the estimate of k and F(t) as given constants, $\ln(\theta)$ can be calculated by

$$\ln(\theta) = \ln(t) - \frac{1}{k}\ln(\ln(\frac{1}{1-F(t)}))$$

Since θ is a constant for each set of data, the value of $\ln(\theta)$ is constant for each t_i which leads to the following:

$$\sum_{i=1}^{n}\ln(\theta) = \sum_{i=1}^{n}[\ln(t_i) - \frac{1}{k}\ln(\ln(\frac{1}{1-F(t_i)}))]$$

$$n \ln(\theta) = \sum_{i=1}^{n} \ln(t_i) - \frac{1}{k} \sum_{i=1}^{n} \ln\left(\ln\left(\frac{1}{1-F(t_i)}\right)\right)$$

$$\ln(\theta) = \left(\sum_{i=1}^{n} \ln(t_i) - \frac{1}{k} \sum_{i=1}^{n} \ln\left(\ln\left(\frac{1}{1-F(t_i)}\right)\right)\right)/n$$

The value of $\ln(\theta)$ can be used to estimate $\ln(t_i)$ and these estimated values compared to the actual values of $\ln(t_i)$ observed for one estimated linear relationship. If various values of c are subtracted from the sample data, each new sample can be used to estimate k and θ and the parameters which provide the best fit to a linear relationship accepted. The method allows for an estimate of $c=t_i$ by using t_2 and n-1 instead of t_1 and n respectively wherever required. In this case, t_1 is effectively censored.

The step-by-step procedure for this method is as follows:

1. Generate the abscissa values, x_i, from $\ln\left(\ln\left(\frac{1}{1-F(t_i)}\right)\right)$ using median ranks as the plotting position of $F(t_i)$.

2. Calculate the $\sum_{i=1}^{n} x_i$

3. In a loop ranging from J = 0 to 10 do the following:

   a. set $\hat{c}_j = (J)\ (0.1)\ (t_i)$

   b. calculate the y_i by the relationship $y_i = \ln(t_i - \hat{c}_j)$

   c. calculate $\sum_{i=1}^{n} y_i$

   d. set $k_j = \frac{x_n - x_1}{y_n - y_1}$

   e. calculate $\ln(\theta)$ by $\ln(\theta_j) = \left(\sum_{i=1}^{n} y_i - \frac{1}{k} \sum_{i=1}^{n} x_i\right)/n$

   f. calculate the natural logarithm of the estimated failure times, $E(y_i)$, by $E(y_i) = \frac{1}{k_j} x_i + \ln(\theta_j)$

   g. calculate the sum of squares of errors, SSE_j, by $SSE_j = \sum_{i=1}^{n} (E(y_i) - y_i)^2$

4. Determine the smallest SSE_j and use $\hat{c}=c_j$, $\hat{k}=k_j$, $\hat{\theta}=\exp(\ln(\theta_j))$

The method of estimating k from the slope calculated with the

extreme values could result in some degradation of the estimate if one (or both) of the extreme values were outliers. Due to their long separation, a considerable deviation would have to occur in one of the extreme values of t before the effect could be expected to be serious. However, to avoid any potential problem of this nature, or others pointed out further on, it is only prudent to examine the data before entering it into the computer for analysis. Because of the method of obtaining linear relationships and checking them by best least squares fit, this method will be referred to as the Linear Least Squares (L.L.S.) method of parameter estimation. Appendix B contains the program for this method of parameter estimation.

Since the L.L.S. method depends to some extent on the plotting position used, various plotting positions were tested against several parameter combinations similar to those selected for the Double Monte Carlo procedure developed. The plotting positions tested were:

1. the mean, $j/(n+1)$;
2. median, $(j-.3)/(n+.4)$;
3. $(j-.375)/(n+.25)$;
4. midpoint, $(j-.5)/n$; and
5. the mode, $(j-1)/(n-1)$.

The median rank method was selected since it gave consistently closer estimates than the other two methods and also it has been extensively used with good results.

Initial LLS Parameter Estimation Tests

Three manual plots of twelve data points each on Weibull paper were made as a check for gross errors in the LLS method. For the first check,

parameters of c=10, k=2.3 and θ=120 were used to generate the data points. The LLS routine provided estimates of $\hat{c}$=40.3, $\hat{k}$=1.48, and $\hat{\theta}$=105. The plot of the data points with $\hat{c}$=0 (•) and $\hat{c}$=40.3 (⊙) is provided in Figure 3. The estimate selected by the program appears to plot accurately on the Weibull chart. For the second and third checks, the data points corrected for $\hat{c}$ which the program selected were plotted and the parameters estimated manually. In both of these cases, the data plotted well and there was no appreciable difference in the plotted parameter estimates. Parameters used to generate the data were c=30, k=2.3, θ=180, and c=30, k=3.1 and θ=400. The respective parameter estimates were $\hat{c}$=81, $\hat{k}$=1.35, $\hat{\theta}$=158 and $\hat{c}$=162, $\hat{k}$=1.76, $\hat{\theta}$=302. The plots of the estimates are provided in Figures 4 and 5.

The LLS method of parameter estimation was then tested by the following computerized procedure.

1. Generate random samples of size 5 to 50 from three-parameter Weibull distributions with pre-selected parameters.

2. Calculate true system reliability from the pre-selected parameters. T was arbitrarily set equal to 100 throughout.

3. Estimate the parameters from the sample data generated.

4. Calculate the estimated reliability, $\hat{R}(100)$, from the estimated parameters. If $\hat{c}$>100, set $\hat{R}(100)$=1.

5. Three sets of parameters were each used to generate 5, 10, 20, and 50 random samples for fifty parameter estimations at each of the twelve combinations.

6. The mean square error between the estimated reliabilities and the true reliability was calculated for each combination.

The true parameters used for the test, true reliabilities, number of

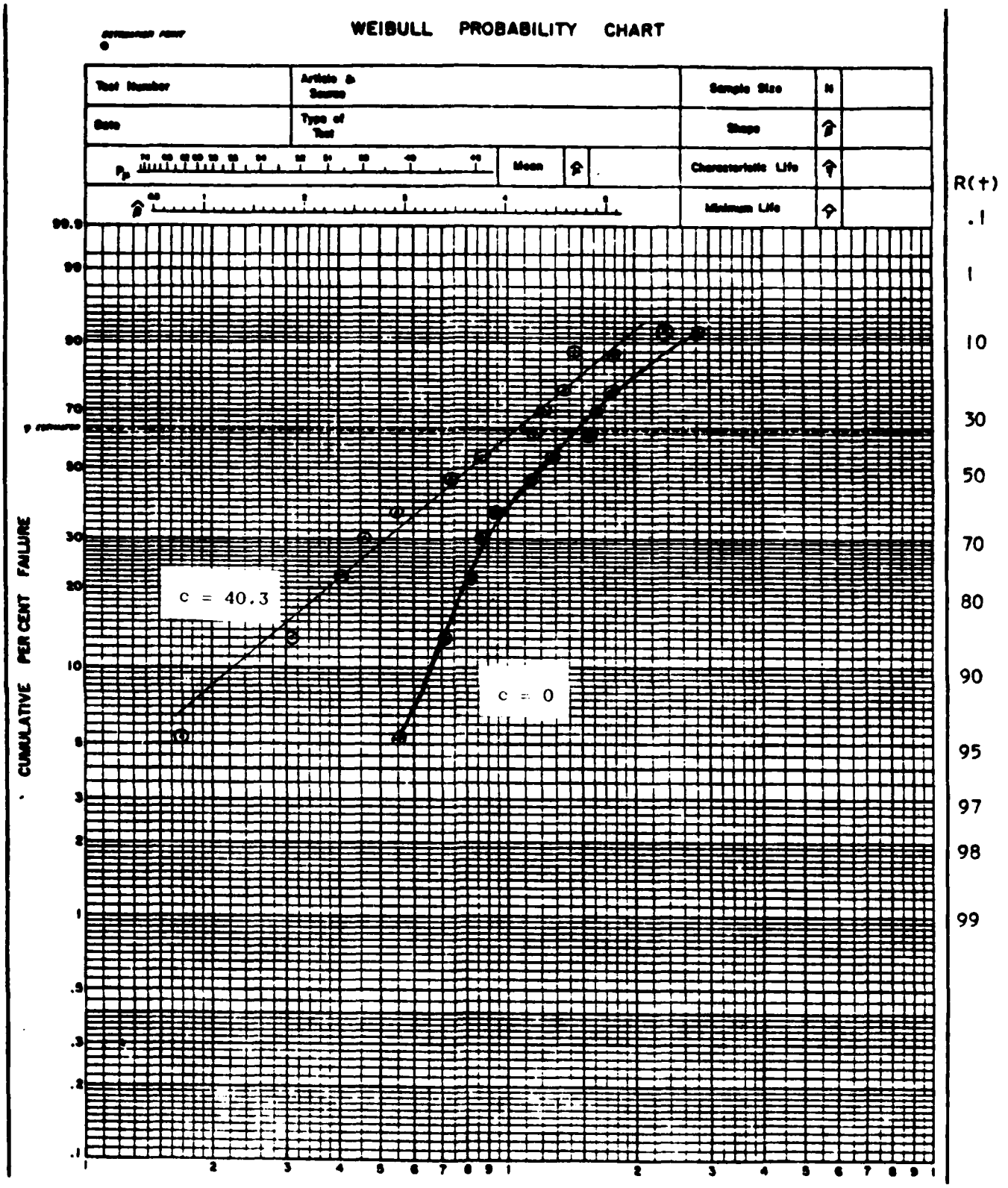

Fig. 3 L.L.S. Plot 1

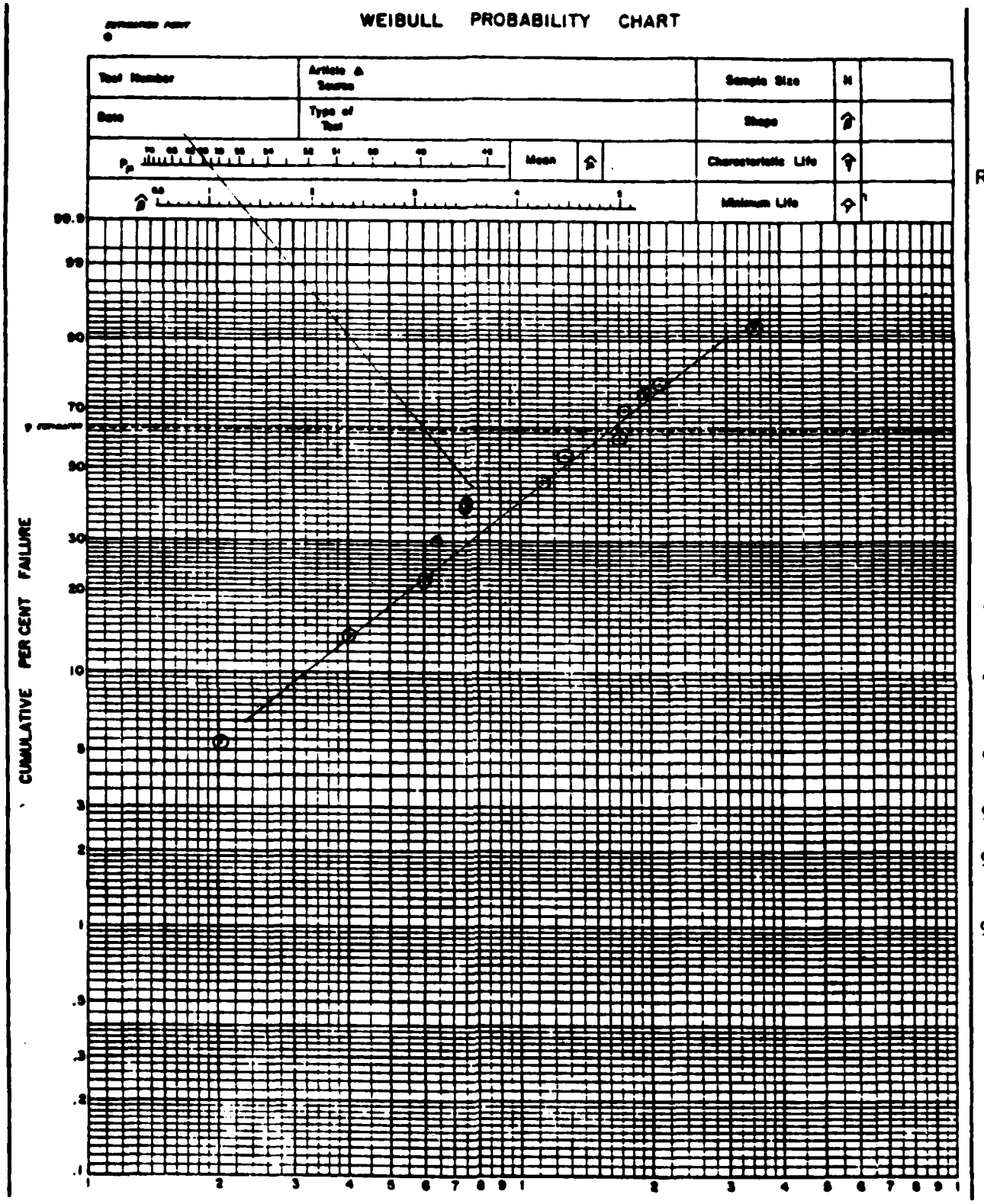

Fig 4. L.L.S. Plot 2

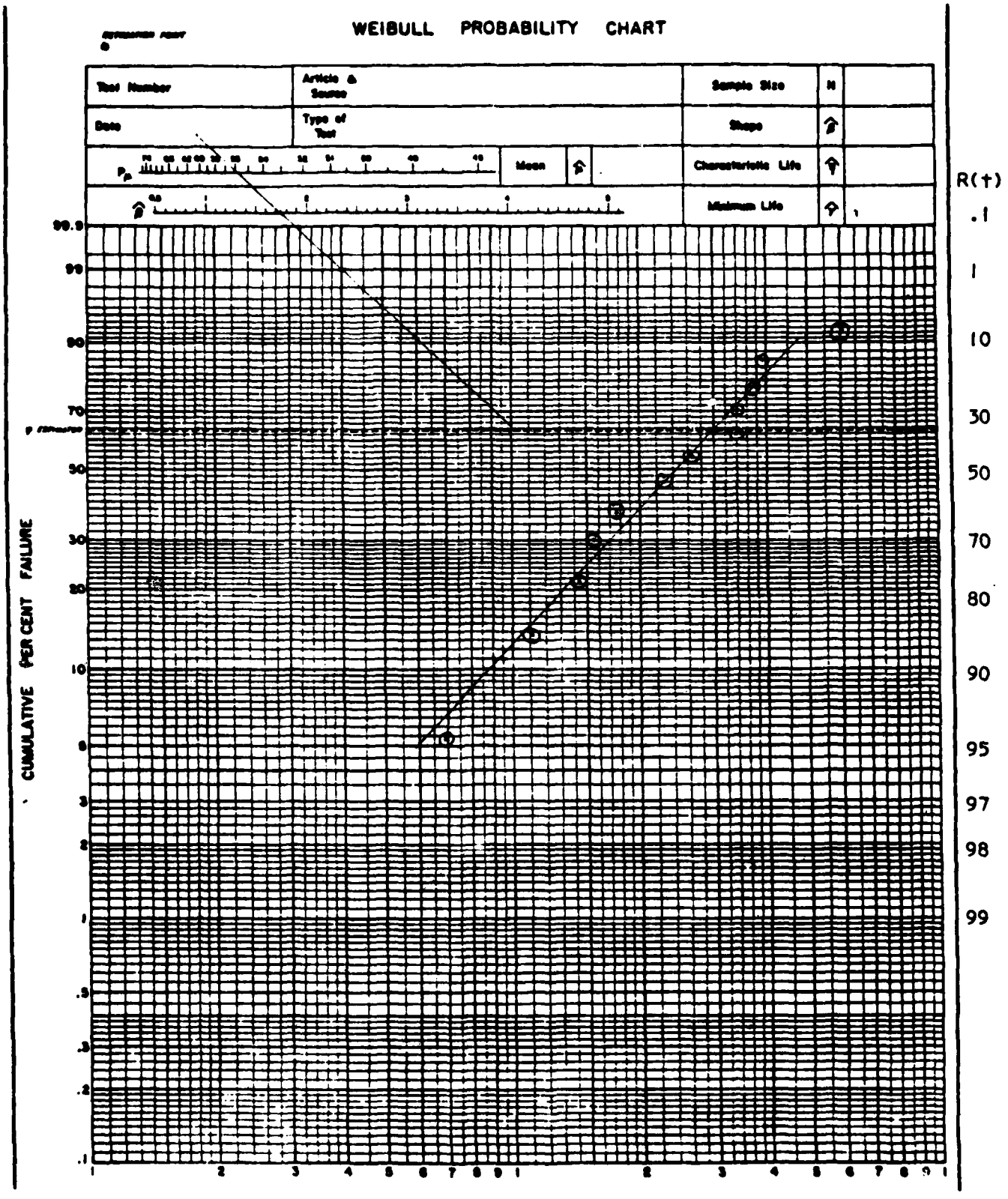

Fig 5. L.L.S. Plot 3

time $\hat{R}(100)=1$, and mean square errors for reliability are listed in Table II. A graphical illustration of the method from ten random samples is provided in Figure 6. For this illustration, the samples were 105.3, 133.6, 196.4, 233.6, 360, 365, 372.1, 417.2, 428.5, and 566.8 from true parameters of c=25, k=2.1, and e=280 with a true reliability of 0.939.

The parameters obtained from the linear least squares method fit the estimated cumulative distribution function to the true cumulative distribution function very well. However, the estimated parameter sets generally had a $\hat{c}$ that was low, and a $\hat{k}$ and $\hat{e}$ that were high. For example, with c=50, k=2 and e=280 and a sample size of ten, the average estimated parameters were $\hat{c}$=3.84, $\hat{k}$=2.86 and $\hat{e}$=331. Figure 7 illustrates a cumulative distribution obtained from the true parameters and the average estimated parameters. This tendency to underestimate c and overestimate k and e was evident for all the initial sample size and parameters tested.

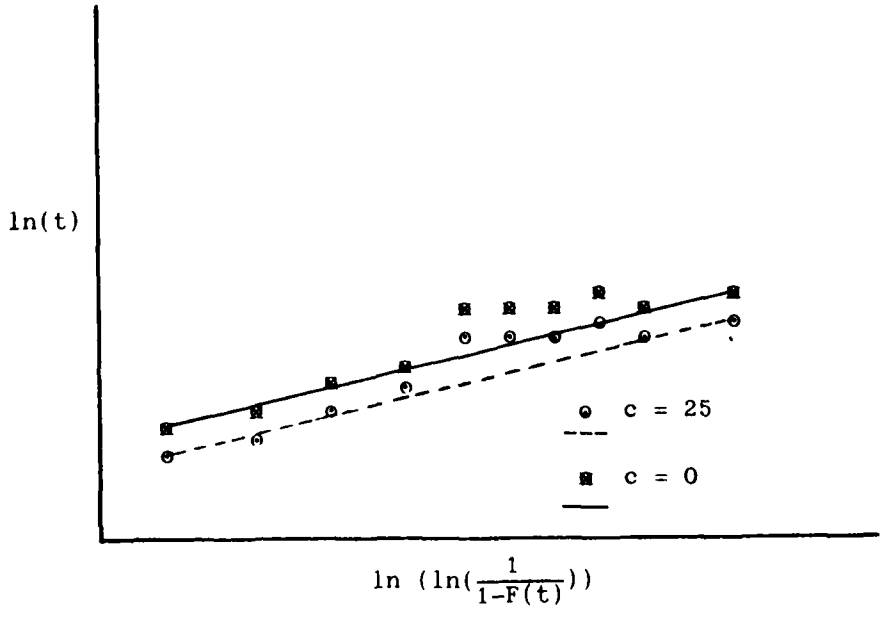

Fig 6. Illustration of Graphical Method

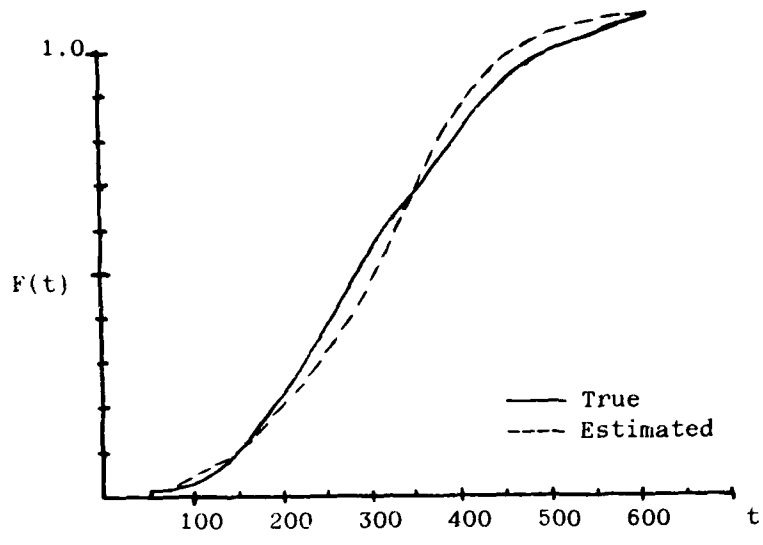

Fig 7. True and Estimated c.d.f.s

TABLE II

MSE Reliability Test of L.L.S Method

Sample Size	MSE	R(100)=1	c	k	e	R(100)
5	.0234	0	10	2.8	140	.7481
10	.0093	0				
20	.0063	0				
50	.0035	0				
5	.0206	1	0	1.1	400	.8044
10	.0083	0				
20	.0052	0				
50	.0032	0				
5	.0033	3	50	2.0	280	.9686
10	.0009	1				
20	.0008	22				
50	.0002	0				

As a further test of the L.L.S. method, a comparison of individual parameter MSEs was made with the results of Miller (Ref 18) using the parameters c=10, e=1 and k ranging from .5 to 4.0. The results with these parameters were poor so the test was discontinued for further analysis of the preliminary findings. An attempt was made to plot the data points on Weibull paper but this proved infeasible. With the value of c extremely high compared to e, all of the points effectively plotted together. Any least squares errors from the linear relationship under these conditions becomes rather meaningless. When the data is run on a computer program, the true best estimate was close to the first order statistic, but because of the clustering of data points with c close to 0, the computer would often select a low value of c with a resulting high value of e. Table III is an example of the data points evaluated on one run of the L.L.S. program. The points are values of failure times (t) minus c over a range in c from c=0 to c=t(1) in increments of 1/10 of the first order statistic, t(1). For this example, the following values applied:

sample size = 12

c = 10

k = 2

e = 1

MSE c = 62.9

MSE k = 207

MSE e = 66

TABLE III

Example of Data for L.L.S. Test

c=0	10.34	10.46	10.56	10.59	10.67	10.86
	10.98	11.27	11.29	11.44	11.52	12.56
c=.1 t(1)	9.31	9.43	9.52	9.56	9.63	9.82
	9.95	10.23	10.26	10.41	10.48	11.53
c=.2 t(1)	8.27	8.39	8.49	8.52	8.60	8.79
	8.91	9.20	9.22	9.38	9.45	10.49
c=.3 t(1)	7.24	7.36	7.45	7.49	7.56	7.75
	7.88	8.16	8.19	8.34	8.41	9.46
c=.4 t(1)	6.20	6.32	6.42	6.45	6.53	6.72
	6.84	7.13	7.15	7.31	7.38	8.43
c=.5 t(1)	5.17	5.29	5.38	5.42	5.49	5.68
	5.81	6.10	6.12	6.27	6.34	7.39
c=.6 t(1)	4.13	4.25	4.35	4.38	4.46	4.65
	4.77	5.06	5.09	5.24	5.31	6.36
c=.7 t(1)	3.10	3.22	3.32	3.35	3.43	3.62
	3.74	4.03	4.05	4.20	4.27	5.32
c=.8 t(1)	2.06	2.18	2.28	2.31	2.39	2.58
	2.70	2.99	3.02	3.17	3.24	4.29
c=.9 t(1)	1.03	1.15	1.25	1.28	1.36	1.55
	1.67	1.96	1.98	2.13	2.21	3.25
c=t(1)		.120	.216	.250	.326	.517
	.640	.928	.952	1.10	1.17	2.22

The second method developed was actually derived from the L.L.S. method. In order to avoid the problems caused by a large c relative to e, c was estimated from a linear extrapolation of the first two order statistics. This eliminated the time consuming process of selecting the best fit to a linear relationship, so k was estimated by the average slope between order statistics. e was estimated as before using the values of c, k, and the order statistics. This modified L.L.S. method

proved to be extremely fast in the parameter estimation comparison tests described later. The computer routine for this modified L.L.S. (M.L.L.S.) is included in Appendix B.

The third new method of parameter estimation developed used the same procedure of extrapolating from the first two order statistics to estimate c. Once c is known, k and e can be quickly estimated by maximum likelihood. These estimators of k and e are:

$$\hat{k} = n/\{([n \sum_{i=1}^{n} x_i^{\hat{k}} \ln(x_i)]/\sum_{i=1}^{n} x_i^{\hat{k}}) - \sum_{i=1}^{n} \ln(x_i)\}$$

$$\hat{e} = [(\sum_{i=1}^{n} x_i^{\hat{k}})/n]^{1/\hat{k}}$$

An iterative routine was used to find $\hat{k}$ where $x_i = t_i - \hat{c}$. The estimate of k was then used directly to estimate e. This method is referred to as the Modified Maximum Likelihood (M.M.L.) method. The computer routine for the M.M.L. method is included in Appendix B. Theoretical development of these estimators of k and e is included in Appendix C.

Parameter Estimation Tests

The modified L.L.S. method and the M.M.L. were tested for individual parameter MSEs using c=10, e=1, and k from 0.5 to 4.0. The results of these tests, listed in Table IV, are comparable to those obtained by Miller (Ref 18) for these parameter selections. However, for this project the accuracies of estimation of the parameters of more widely spread distributions were required. Therefore, the component parameters previously selected for an 85% system reliability were used to test the L.L.S., the modified L.L.S., the L.L.S. using an average slope for k instead of just the extreme values, and the M.M.L. Sample sizes of 5, 10, 20, and 50 were selected to provide a wide range without an excessive number of points. All of the above tests consisted of calculating the

TABLE IV

Parameter Estimation MSEs for c=10, θ=1

Sample Size

		4	8	12	16	20
c	M.L.L.S.	.188	.010	.002	.001	.000
	M.M.L.	.188	.010	.002	.001	.000
k	M.L.L.S.	.335	.058	.031	.017	.012
	M.M.L.	.828	.094	.037	.021	.014
θ	M.L.L.S.	4.40	1.05	.542	.360	.264
	M.M.L.	3.82	.963	.622	.432	.314

k = 0.5

Sample Size

		4	8	12	16	20
c	M.L.L.S.	.101	.027	.008	.006	.004
	M.M.L.	.101	.028	.011	.007	.004
k	M.L.L.S.	.344	.153	.082	.069	.052
	M.M.L.	1.01	.308	.117	.070	.053
θ	M.L.L.S.	.440	.175	.137	.080	.064
	M.M.L.	.415	.163	.102	.084	.061

k = 1.0

Sample Size

		4	8	12	16	20
c	M.L.L.S.	.156	.086	.059	.044	.036
	M.M.L.	.156	.087	.056	.045	.037
k	M.L.L.S.	.759	.668	.582	.472	.414
	M.M.L.	.869	.695	.438	.350	.299
θ	M.L.L.S.	.218	.139	.104	.079	.068
	M.M.L.	.228	.137	.093	.075	.064

k = 2.0

Sample Size

		4	8	12	16	20
c	M.L.L.S.	.241	.163	.128	.105	.920
	M.M.L.	.241	.163	.123	.106	.085
k	M.L.L.S.	2.69	2.33	2.08	1.80	1.65
	M.M.L.	1.78	1.84	1.52	1.35	1.20
θ	M.L.L.S.	.287	.210	.173	.141	.125
	M.M.L.	.298	.209	.159	.134	.110

k = 3.0

Sample Size

		4	8	12	16	20
c	M.L.L.S.	.321	.238	.213	.171	.155
	M.M.L.	.321	.237	.194	.172	.151
k	M.L.L.S.	6.45	5.59	6.04	4.52	4.21
	M.M.L.	4.35	4.35	3.88	3.56	3.26
θ	M.L.L.S.	.365	.285	.265	.210	.191
	M.M.L.	.375	.283	.231	.201	.177

k = 4.0

TABLE V

Parameter Estimation MSEs for System Components

Sample Size

		5	10	20	50
L.L.S.	c	1031*	672*	713	625
	k	1.12	.350	.135	.343
	θ	10,959*	6169	4155	1436
L.L.S.	c	1046	744	640	625
Average	k	.976	.245	.078	.086
Slope	θ	11,084	5869*	2986*	2645
Modified	c	4761	1589	529*	99*
L.L.S.	k	.285*	.185*	.076	.030
	θ	14,309	7104	3490	1176
M.M.L.	c	4761	1589	529*	99*
	k	.630	.260	.069*	.026*
	θ	14,084	6915	3352	1130*

c = 25 k = 1.2 θ = 250

Sample Size

		5	10	20	50
L.L.S.	c	508*	360*	1502	225*
	k	2.62	.791	.611	.312
	θ	2204*	1285*	1138	427*
L.L.S.	c	579	405	1447	225
Average	k	2.28	.582*	.430	.123*
Slope	θ	2338	1393	1123*	514
Modified	c	4317	2225	1265*	423
L.L.S.	k	.763*	.618	.409	.189
	θ	6317	3766	2280	820
M.M.L.	c	4317	2225	1265*	423
	k	.801	.594	.307*	.134
	θ	6500	3688	2154	761

c = 15 k = 2 θ = 180

Sample Size

		5	10	20	50
L.L.S.	c	1944*	610*	394	0*
	k	.405	.093	.031	.026
	θ	116,292	56,424	44,906	12,354
L.L.S.	c	2280	802	351	0*
Average	k	.360	.074*	.022*	.009*
Slope	θ	116,892	51,187	35,276	8,318
Modified	c	13,999	2,661	398	39.28
L.L.S.	k	.228*	.096	.028	.011
	θ	120,366	50,150	23,266	8,152
M.M.L.	c	13,999	2,661	398	39.28
	k	.538	.148	.031	.001
	θ	111,635*	48,043*	22,712*	8,002*

c = 0 k = .8 θ = 470

Sample Size

		5	10	20	50
L.L.S.	c	553*	320*	2.565	100*
	k	3.08	.891	.921	.259
	θ	2087*	1125*	1779*	318
L.L.S.	c	593	386	2502*	100
Average	k	2.68	.667*	.695	.102*
Slope	θ	2181	1290	1923	311*
Modified	c	6339	3557	2205	839
L.L.S.	k	1.16	.938	.670	.329
	θ	8568	5365	3460	1360
M.M.L.	c	6339	3557	2205	839
	k	.995*	.823	.501*	.235
	θ	8812	5256	3281	1268

c = 10 k = 2.3 θ = 200

Sample Size

		5	10	20	50
L.L.S.	c	959*	906	2152	900*
	k	11.4	4.30	3.83	2.47
	θ	1173*	1037*	1701*	944*
L.L.S.	c	996	905*	2120	900
Average	k	9.83	3.15	2.92	1.36
Slope	θ	1191	1068	1850	1013
Modified	c	3722	2545	1880	980
L.L.S.	k	4.25	3.47	2.81	1.67
	θ	4455	3184	2375	1231
M.M.L.	c	3722	2572	1775*	1016
	k	3.04*	2.80*	2.17*	1.32*
	θ	4536	3191	2191	1209

c = 30 k = 3.5 θ = 120

Sample Size

		5	10	20	50
L.L.S.	c	2428	2402	1385	2500
	k	5.16	2.33	.923	2.25
	θ	3816*	3350	1125	2686
L.L.S.	c	2425*	2322	1264	2500
Average	k	4.43	1.63	.539	.959
Slope	θ	3931	3441	1083*	3343
Modified	c	2998	1545	879	294*
L.L.S.	k	.763*	.618	.409	.189
	θ	4387	2615	1583	570
M.M.L.	c	2298	1536*	813*	309
	k	.801	.586*	.319*	.134*
	θ	4514	2571*	1429	542*

c = 50 k = 2.0 θ = 150

MSEs based on Monte Carlo runs of 1000 repetitions. The results of the estimations are listed in Table V with the best parameter estimate for each sample size marked by an *. The computer CPU times required for 1000 runs of the four sample sizes for each method are listed in Table VI.

TABLE VI
Parameter Test CPU Times

Method	Time in Seconds
L.L.S.	76
Modified L.L.S.	22
L.L.S. Average Slope	90
M.M.L.	46

For comparison, a limited test of the Harter-Moore method of maximum likelihood program was run. On the initial runs using 1000 Monte Carlo repetitions, c=0, k=0.8, and e=470, only the runs for sample sizes 5 and 20 converged within 400 seconds so no further runs of 1000 repetitions were made. The MSEs from these two runs are listed in Table VII.

TABLE VII
Harter-Moore Method - 1000 Repetitions

Sample Size	MSE c	MSE k	MSE e
5	9013	.3978	73574
20	342	.025	17178

The number of repetition was decreased to 100 and a further test was run using sample sizes of 5, 10, 20, and 50, c=25, k=1.2, and e=250. The run at sample size 10 took 70 seconds which was proportional to the length of time taken for the other runs. The MSEs of this test are listed in Table VIII.

TABLE VIII

Harter-Moore Method - 100 Repetitions

	Sample Size			
	5	10	20	50
MSE c	4100	1238	553	156
MSE k	1.173	.329	.092	.033
MSE o	13533	5947	2735	959

$c = 25 \quad k = 1.2 \quad \theta = 250$

Discussion of Parameter Estimation Test

On the basis of the tests using c=10 and θ=1 there is little difference between the modified L.L.S. and the M.M.L. in accuracy, with the modified L.L.S. being twice as fast. The estimates of c are the same for both, with the occasional small differences attributable to different random number seed values, since they use the same method of estimating c. Both methods give reasonably good results and both are easy to implement. The L.L.S. method does not produce satisfactory results with a large c relative to θ. If the data is all clustered about one point, then some method must be used to spread it for analysis such as estimating a location parameter from the first two order statistics or subtracting a large fraction of the first order statistic from all the data.

Using data generated from more widely spread parameters, the L.L.S. appears very good. It is more consistent in its estimates of c and the estimate of k and θ are as good or close to as good as the estimates of any of the other methods. It even compares favorably with the well established Harter-Moore method of three parameter maximum likelihood estimation, particularly for small sample sizes. Of interest is that the

L.L.S. generally provided better estimates than the L.L.S. modified to use an average slope to estimate k. The M.M.L. method was more consistent in its estimates of k than any of the other methods. A simple count of the number of parameters most accurately estimated shows the L.L.S. and the M.M.L. method tied at 25 each with the L.L.S. better at estimating c and the M.M.L. better at estimating k. Neither parameter can be considered the more critical because the importance of an error in either depends on their magnitude and the relative magnitudes of c and e. The selection of the best method depends on which produces the best results in the Double Monte Carlo procedure. Given equivalent results, the fastest method, M.M.L., would be preferable.

The results of the tests on parameter estimation methods indicate strongly that extensive trials with a wide range of combinations of the three parameters are necessary before any method can be selected as the universal "best" or even as the best for a particular purpose. Until this is done, it is advisable to visually inspect the data prior to any furhter analysis. If the data appears well spread, the L.L.S. would be a good choice of method to estimate the parameters. If the data appears to be bunched in one small range, then the M.M.L. would likely be preferable. Also, as for any other use of data, the data should be checked for outliers before parameter estimation, and for goodness of fit after parameter estimation.

The original systems reliability confidence level tests were selected for sample sizes ranging from ten to one hundred. These sizes were selected primarily because of the need for relatively large sample sizes to obtain consistent results with the maximum likelihood method of parameter estimation. However, the results obtained from the L.L.S. and

the M.M.L. methods developed appear reasonable for all the sample sizes tested: 5, 10, 20 and 50. Therefore, the project was modified to use these sample sizes rather than the sample sizes originally picked.

To select the better method and to illustrate the use of the program, 1000 Monte Carlo runs were divided into seven sets of "real" data for testing at each of the system reliabilities 74%, 85%, and 96%. Each of these sets used the real data once but generated simulated failures and confidence limits 1000 times. Also checked on these runs was the number of times the estimate of c was greater than the selected mission time of 100 units. For the L.L.S., the largest percent of times was about 5% compared to about 22% for the M.M.L. The overall results for the L.L.S. method were much better than the results for the M.M.L. method at 85% system reliability, so the M.M.L. method was not tested further. For this project, the L.L.S. method of parameter estimation was selected as the best. The average results of these test runs are listed in Table IX.

TABLE IX

Parameter Estimation Selection Results

L.L.S. Method

R = 74%

Confidence Level							Sample Size
.50	.60	.70	.80	.90	.95	.99	
.297	.387	.662	.853	.975	1.0	1.0	5
.418	.499	.460	.622	.793	.899	.993	10
.360	.396	.443	.640	.885	.983	1.0	20
.345	.478	.653	.782	.851	.911	.995	50

R = 85%

Confidence Level							Sample Size
.50	.60	. 70	.80	.90	.95	.99	
.313	.428	.729	.904	.998	1.0	1.0	5
.445	.450	.473	.628	.850	.962	1.0	10
.280	.380	.441	.541	.589	.785	.978	20
.335	.598	.707	.804	.875	.975	1.0	50

R = 96%

Confidence Level							Sample Size
.50	.60	.70	.80	.90	.95	.99	
.345	.490	.759	.977	1.0	1.0	1.0	5
.449	.450	.456	.608	.948	.998	1.0	10
.197	.249	.251	.268	.511	.765	.921	20
.490	.608	.741	.843	.952	.997	1.0	50

M.M.L. Method

R = 85%

Confidence Level							Sample Size
.50	.60	.70	.80	.90	.95	.99	
.300	.300	.300	.300	.354	.448	.635	5
.393	.448	.450	.450	.454	.518	.712	10
.287	.386	.533	.647	.647	.647	.739	20
.305	.496	.604	.745	.845	.852	.918	50

V Assessment of Method

Results

The three-parameter Double Monte Carlo Method was run for 1000 repetitions at system reliabilities of 74%, 85%, and 96% with component failure sample sizes of five, ten, twenty, and fifty. Initially, seventy-five points were used for the component sample distributions of reliability estimates to be compatible with previous work (Ref 13). The results of these runs are listed in Table X. The 1000 repetitions of each combination were divided into six runs of 150 and one run of 100 to keep all required CPU times below 4000 seconds.

In order to check the sensitivity of the Double Monte Carlo method to the number of points in the component sample distributions of reliability estimates, the number of points was increased to 150 and the method used for a system reliability of 85% with component failure sample sizes of five, ten, and twenty. The results of these runs are listed in Table XI.

Since increasing the number of points in the component sample distributions of reliability estimates did not increase the accuracy of the method, the number of points was decreased to fifty and the Double Monte Carlo Method was checked at reliabilities of 74%, 85%, and 96% with component failure sample sizes of five, ten, and twenty. The reduction in the number of points from seventy-five to fifty resulted in a 1/3 reduction in the number of parameter estimations required. Since parameter estimation takes nearly all of the computer CPU time for the method, this reduction in points also resulted in about a 1/3 reduction in CPU time. This, combined with limiting the failure sample sizes to

TABLE X

Double Monte Carlo Results: Distribution Size 75

R = 74%

Confidence Level							Sample Size
.50	.60	.70	.80	.90	.95	.99	
.447	.550	.649	.740	.823	.874	.944	5
.475	.576	.674	.765	.855	.914	.966	10
.404	.519	.603	.689	.795	.877	.942	20
.488	.594	.695	.797	.907	.952	.979	50

R = 85%

Confidence Level							Sample Size
.50	.60	.70	.80	.90	.95	.99	
.420	.528	.642	.729	.817	.876	.940	5
.470	.578	.675	.757	.848	.910	.976	10
.362	.455	.522	.609	.715	.786	.856	20
.522	.628	.730	.842	.933	.967	.989	50

R = 96%

Confidence Level							Sample Size
.50	.60	.70	.80	.90	.95	.99	
.371	.501	.607	.714	.807	.872	.934	5
.578	.687	.786	.863	.935	.959	.989	10
.111	.150	.201	.290	.445	.570	.763	20
.566	.660	.767	.862	.948	.973	.992	50

TABLE XI

Double Monte Carlo Results: Distribution Size 150

R = 85%

Confidence Level							Sample Size
.50	.60	.70	.80	.90	.95	.99	
.415	.522	.623	.730	.824	.879	.933	5
.495	.580	.690	.767	.856	.900	.958	10
.405	.486	.570	.663	.743	.802	.880	20

20, allowed an increase to 500 repetitions within 4000 seconds of CPU time. The results of these runs are listed in Table XII.

The results indicate that a sample size of twenty had the greatest bias of the four sample sizes tested. This was particularly noticeable at a system reliability of 96%. To obtain a better assessment of the bias trend, the Double Monte Carlo method was run for a system reliability of 96% at sample sizes of fifteen, twenty-five, and thirty. Fifty points were used for the sample distributions of reliabilities. The results of these runs are combined with the results of the runs under the same conditions except with failure sample sizes of five, ten, and twenty in Table XIII to show the trend in the system reliability bias.

Discussion of Results

Parameter Estimation. The accuracy of the Double Monte Carlo method is dependent on the accuracy of parameter estimation. In order to be practical for computerized operation, the parameter estimation method used must also be fast. If the bias of the reliability estimates is known, these estimates can be corrected for bias in the program. Depending on the parameter estimation technique used, the bias is not necessarily directly related to sample size. In the case of the L.L.S. method, the routine is fast and reasonably accurate but has no particular optimum properties; therefore, increasing sample size does not imply that the bias will decrease. The results indicate that the system bias becomes increasingly negative as the sample size increases from five to twenty, then increases to slightly positive for a sample size of thirty (Table XIII). Generally, the bias will not be known and it will prove more practical to either establish a system bias or use the confidence

TABLE XII

Double Monte Carlo Results: Distribution Size 50

R = 85%

Confidence Level							Sample Size
.50	.60	.70	.80	.90	.95	.99	
.452	.552	.627	.721	.817	.877	.933	5
.464	.561	.655	.758	.855	.913	.958	10
.396	.481	.556	.659	.757	.816	.890	20

R = 96%

Confidence Level							Sample Size
.50	.60	.70	.80	.90	.95	.99	
.425	.514	.611	.717	.806	.874	.924	5
.453	.571	.654	.754	.860	.914	.958	10
.138	.186	.241	.333	.482	.622	.783	20

R = 74%

Confidence Level							Sample Size
.50	.60	.70	.80	.90	.95	.99	
.475	.570	.648	.738	.825	.882	.932	5
.472	.563	.664	.755	.856	.912	.957	10
.461	.536	.628	.725	.831	.901	.942	20

TABLE XIII

Extended Double Monte Carlo Results: Distribution Size 50

R = 96%

Confidence Level							Sample Size
.50	.60	.70	.80	.90	.95	.99	
.425	.514	.611	.717	.806	.874	.924	5
.453	.571	.654	.754	.860	.914	.958	10
.284	.366	.460	.558	.669	.761	.851	15
.138	.186	.241	.333	.482	.622	.783	20
.558	.669	.749	.833	.928	.964	.988	25
.554	.635	.733	.822	.925	.962	.994	30

level calculated that is closest to the desired level. Since with real data the exact actual parameters will be unknown, the bias or levels used must be for parameters close to the true parameters. This bias can be calculated in the following manner:

1. From the available failure data, estimate the component parameters as accurately as possible. Computer CPU time is not a factor for this estimation.

2. Use these accurate estimates as true parameters and run a Double Monte Carlo program to determine accuracy at the significance level(s) desired. Empirically determine system bias by applying bias to the system reliability estimates until the results are accurate. This bias will be accurate for the estimated parameters (step 1) and conditions and should be close for the true parameters.

3. Use the bias (from step 2) to calculate the appropriate confidence limit(s).

Sample Distribution Size. Three sizes of sample distributions of component reliability estimates (50, 75, 150) were tested with no apparent difference in accuracy of results. The actual error caused by using a small number of points cannot be directly calculated since the distribution of the reliability estimates is not known exactly, but since the error is the difference between the point on the true curve and the point obtained by interpolation between an upper and lower point on the true curve, the error will be much less than the spacing of the points used for establishing the reliability distribution. The use of 50 points gave good results in these trials and should provide the required accuracy for any application.

General Method. The Double Monte Carlo method of confidence limit

estimation is easy to implement and provides good consistent results for the three-parameter Weibull. There is a requirement to remove bias for maximum accuracy, but an estimate of bias is readily obtainable and easy to apply. If the extra accuracy is not required, then considerable computer time can be saved by using the biased estimators for approximate confidence limits. Four parameter estimation methods are included in Appendix B, but the method can be readily used with any parameter estimation technique. In the results of this development, the estimation of the location parameter appeared to be the most critical; therefore, the data should be examined for a gross estimate of this parameter before being input into any particular routine. A good approach would be to ensure the data is well spread by removing some portion of the first order statistic; then use the L.L.S. method of parameter estimation.

VI Illustration of Method

In order to illustrate the method, the component parameters used for the system reliability of 0.96 were used to generate one set of fifteen data points for each component using a seed value of 17943. These data are included in Appendix D. The L.L.S. method was used to estimate the parameters from these data and a Double Monte Carlo program was run, using these estimated parameters as true parameters, to calculate system bias. As a matter of interest, the Harter-Moore three-parameter maximum likelihood routine was also run to estimate the parameters from these data points. The results of the parameter estimation routines are listed in Table XIV.

A system bias of -0.005 was found to provide good results at the high confidence limits of interest using seed values of 7539 and 96 for the random number generation on two Double Monte Carlo runs of 500 each for 1000 total runs. As a test of the accuracy of this bias, it was applied to the system reliability of 1000 Double Monte Carlo runs using the known true parameters from which the failure data were generated and seed values of 135 and 17. Table XV lists the results of these two runs and the original results not corrected for bias.

TABLE XIV

Parameters for Demonstration

Component	Method	c	k	θ
1	L.L.S.	69.5	2.42	226
	True	10	2.9	270
	H-M M.L.	118	1.70	173
2	L.L.S.	56.5	1.49	458
	True	0	2.1	470
	H-M M.L.	82.2	1.70	400
3	L.L.S.	82.5	2.36	333
	True	15	2.3	400
	H-M M.L.	.0000027	3.63	422
4	L.L.S.	46	3.21	136
	True	30	3.0	160
	H-M M.L.	.0000023	5.54	183
5	L.L.S.	80	2.80	206
	True	25	2.7	250
	H-M M.L.	152	1.59	123
6	L.L.S.	74.9	1.40	229
	True	50	2.0	280
	H-M M.L.	88	1.19	213

TABLE XV

Illustration Results

	Confidence Level						
	.50	.60	.70	.80	.90	.95	.99
Bias Estimates	.615	.704	.781	.849	.920	.949	.974
True Parameters with Bias	.335	.413	.507	.604	.711	.795	.866
True Parameters without Bias	.284	.366	.460	.558	.669	.761	.851

VII Concluding Material

Summary

A Double Monte Carlo method of estimating complex system reliabilities at any confidence level was developed based on three-parameter Weibull component failure distributions. In order to implement the method, three fast, accurate parameter estimation routines were developed and tested. The most effective routine, the L.L.S., was selected and used in the Double Monte Carlo program. Good results were obtained with componet failure sample sizes as low as five for reliabilities from 74% to 96%. A step-by-step procedure with an illustration of the method are provided.

Conclusions

It is concluded that:

1. the Double Monte Carlo method can be readily used to provide reliability confidence limits for complex systems with three-parameter Weibull failure distributions;
2. sample sizes as small as five can be used for system reliability confidence limit estimation;
3. the Double Monte Carlo method is not adversely affected by reducing the number of points in the sample distributions of reliabilities to fifty; and
4. further development, full testing, and establishing biases for three-parameter Weibull reliability estimations is needed.

Recommendations

It is recommended that;

1. further development of computerized reliability estimation routines be done; and

2. bias tables be developed for a fast, accurate three-parameter Weibull reliability estimation routine.

Bibliography

1. Antoon, David F. Confidence Intervals and Tests of Hypothesis of a System Whose Underlying Distribution is a Two-Parameter Weibull. Unpublished Thesis. Wright-Patterson Air Force Base, Ohio: Air Force Institute of Technology, December 1979.

2. Augenstein, B. W., D. Dreyfuss, and A. G. Parish. "Bunched Launch, Bunched Acquisition, and Work-arounds - Elements of Alternative Spacecraft Acquisition Policies". Rand Report R-2166-AF, October, 1979.

3. Bernhoff, Oscar A. Confidence Limits for System Reliability Based on Component Test Data. Unpublished Thesis. Wright-Patterson Air Force Base, Ohio: Air Force Institute of Technology, August 1963.

4. Bilikam, J. Edward and Albert H. Moore. "Estimation of Reliability from Multiple Independent Grouped Censored Samples". IEEE Transactions on Reliability, Vol R-26, 57-60, April 1977.

5. Bilikam, J. E. and A. H. Moore. "Estimation of Reliability from Multiple Independent Grouped Censored Samples with Failure Times Known". IEEE Transactions on Reliability, Vol R-27, 329-331, December 1979.

6. Boardman, T. J. and P. J. Kendall. "Estimators of the Parameters of a Binomial Mixture of Grouped - Censored Exponentials -- Useful in Life Testing Problems". Communications in Statistics, Vol 2 (1), 29-44, 1973.

7. Depuy, Kathleen M. A Comparison of the Accuracy of Univariate and Bivariate Techniques for Finding the Lower Confidence Limits of System Reliability. Unpublished Thesis. Wright-Patterson Air Force Base, Ohio: Air Fore Institute of Technology. December 1980.

8. Easterling, Robert G. "Discussion of: Confidence Interval Estimation for the Weibull and Extreme Value Distributions," Technometrics, 20; 365-368, November 1978.

9. Gatliffe, Thomas R. Accuracy Analysis for the Lower Confidence Limit Procedure for System Reliability. Unpublished Thesis. Monterey, California: Naval Post-graduate School, 1976.

10. Hahn, Gerald J. and Samuel S. Shapiro. Statistical Models in Engineering. John Wiley and Sons, Inc., New York. 1967.

11. Harter, H. Leon and Albert H. Moore. "An Evaluation of Exponential and Weibull Test Plans". IEEE Transactions on Reliability, Vol R-25: 100-104, June 1976.

12. Harter, H. Leon and Albert H. Moore. "Maximum Likelihood Estimation of the Parameters of Gamma and Weibull Population from Complete and from Censored Samples". Technometrics, 7: 639-643, November 1965.

13. Johnston, James W. Jr. A Modified Double Monte Carlo Technique to Approximate Reliability Confidence Limits of Systems with Components Characterized by the Weibull Distributions. Unpublished Thesis. Wright-Patterson Air Force Base, Ohio: Air Force Institute of Technology, December 1980.

14. Kapur, K. C. and L. R. Lamberson. Reliability in Engineering Design. John Wiley and Sons, New York. 1977.

15. Lannon, Robert G. A Monte Carlo Technique for Approximating System Reliability Confidence Limits Using the Weibull Distribution. Unpublished Thesis. Wright-Patterson Air Force Base, Ohio: Air Force Institute of Technology, 1972.

16. Levy, L. L. A Monte Carlo Technique for Obtaining System Reliability Confidence Limits from Component Failure Test Data. Unpublished Thesis. Wright-Patterson Air Force Base, Ohio: Air Force Institute of Technology, 1964.

17. Levy, Louis L. and Albert H. Moore. "A Monte Carlo Technique for Obtaining System Reliability Confidence Limits from Component Test Data". IEEE Transactions on Reliability, R-16, 2:69-72. September 1967.

18. Miller, Robert M. Robust Minimum Distance Estimation of the Three Parameter Weibull Distribution. Unpublished Thesis. Wright-Patterson Air Force Base, Ohio: Air Force Institute of Technology, December 1980.

19. Modesitt, G. E. "Statistical Analysis of Spacecraft Replenishment". Rand Report RM-4739-ARPA, March 1966.

20. Moore, Albert H., J. R. Hobbs, and J. W. Johnston. "A Modified Double Monte Carlo Technique to Approximate Reliability Confidence Limits of Systems". IEEE Transactions on Reliability (submitted April 1982).

21. Moore, Albert H., H. Leon Harter, and Robert C. Snead. "Comparison of Monte Carlo Techniques for Obtaining System-Reliability Confidence Limits". IEEE Transactions of Reliabilty, Vol R-29: 327-332, October 1980.

22. Moore, Albert H., H. Leon Harter, and David F. Antoon. "Confidence Intervals and Tests of Hypothesis for the Reliability of a 2-Parameter Weibull System". IEEE Transactions on Reliability, Vol R-30: 468-470, December 1981.

23. Moore, Albert H. "Extension of Monte Carlo Technique for Obtaining System Reliability Confidence Limits from Component Test Data". Proceedings of National Aerospace Electronics Conference, 459-463. May 1965.

24. Orkand, Donald S. A Monte Carlo Method for Determining Lower Confidence Limits for System Reliability on the Basis of Sample Component Data. Report # ORDBB-VC-4. Dover, NJ: Concepts and Applications Laboratory, Picatinny Arsenal, June 1960. (AD 627 799).

25. Parish, LCol Gary and William Sollfrey. "A Preliminary Analysis of the Effect of Work-Arounds on Space System Performance and Procurement Requirements - A Proposal. Rand Note N-1260-AF. March 1980.

26. Putz, Randall B. A Univariate Monte Carlo Technique to Approximate Reliability Confidence Limits of System with Components Characterized by the Weibull Distribution. Unpublished Thesis. Wright-Patterson Air Force Base, Ohio: Air Force Institute of Technology, December 1979.

27. Quayle, Ronald J. Estimation of the Scale Parameter of the Weibull Probability Density Function by the use of One Order Statistic. Unpublished Thesis. Wright-Patterson Air Force Base, Ohio: Air Force Institute of Technology, August 1963.

28. Rice, Roy C. Incorporation of Asymptotic Normality Properties of the Binomial Distribution Into a Monte Carlo Technique for Estimating Lower Confidence Limits on System Reliability. Unpublished Thesis. Wright-Patterson Air Force Base, Ohio: Air Force Institute of Technology, December 1979.

29. Rosenblatt, J. R. Confidence Limits for the Reliability of Complex Systems. Proceedings of an advanced Seminar conducted by the Mathematics Research Center, US Army, at the Univ of Wisconsin; 115-137, 1962.

30. Snead, Robert C. A Univariate Asymptotic Normal Monte Carlo Method (UANMCM) for Estimating System Reliability Confidence Limits. Unpublished Thesis. Wright-Patterson Air Force Base, Ohio: Air Force Institute of Technology, December 1978.

31. Thoman, Darrel R., Lee J. Bain, and Charles E Antle. "Maximum Likelihood Estimation, Exact Confidence Intervals for Reliability, and Tolerance Limits in the Weibull Distribution". Technometrics, 12:363-370. May 1970.

32. Vandervoot, C. G. "The Effect of Maintainability on the Mission Reliability of Two-Element Redundant Spacecraft Subsystems". Rand Report RM-4824-PR, December 1965.

33. Vonloh, John F. Reliability Growth and its Applications to Dormant Reliability. Unpublished Thesis. Wright-Patterson Air Force Base, Ohio: Air Force Institute of Technology, December 1981.

34. Weibull, W. "A Statistical Distribution of Wide Applicability". Journal of Applied Mechanics, Transactions of ASME, 18:293-297, 1951.

35. Wingo, D. R. "Maximum Likelihood Estimation of the Parameters of the Weibull Distribution by Modified Quasilinearization". IEEE Transactions on Reliability, Vol R-21: 89-93, May 1972.

APPENDIX A. Double Monte Carlo Program

```
      PROGRAM SYSREL
C  THIS PROGRAM PROVIDES CONFIDENCE LIMITS ON THE
C  RELIABILITY OF A COMPLEX SYSTEM WHOSE COMPONENT RELIABILITIES
C  CAN BE ESTIMATED. IT ASSUMES THAT THE UNDERLYING
C  COMPONENT DISTRIBUTIONS ARE 3-PARAMETER WEIBULL
C  AND USES THE DOUBLE MONTE-CARLO TECHNIQUE TO
C  GENERATE RELIABILITY ESTIMATES. INTERNATIONAL
C  MATHEMATICS AND STATISTICS LIBRARY (IMSL) ROUTINES
C  GGUBS,FOR GENERATING A UNIFORM (0,1) RANDOM VARIABLE,
C  AND GGWIB, FOR GENERATING A ONE-PARAMETER WEIBULL
C  RANDOM VARIABLE, ARE USED. A LEAST SQUARES METHOD
C  OF ESTIMATING THE THREE
C  UNKNOWN PARAMETERS IS USED IN ROUTINE PARAM.
C  THE PROGRAM GENERATES ITS OWN REAL FAILURE DATA
C  BUT CAN READILY BE MODIFIED TO USE ACTUAL FAILURE
C  DATA BY DELETING THE APPROPRIATE SECTION.
C  INPUTS:
C    REPS= THE NUMBER OF TIMES SIMULATED RELIABILITIES ARE GENERATED
C          TO ESTABLISH A RELIABILITY DISTRIBUTION
C          FUNCTION.
C    C(I),K(I),THETA(I)= THE PARAMETERS OF THE COMPONENT
C          FAILURE DISTRIBUTIONS.
C    TNFAIL= THE NUMBER OF FAILURES OF EACH COMPONENT
C  ***********
C  OUTPUTS:
C    FOR 50,60,70,80,90,95,99%  THE NUMBER
C    OF TIMES THE LOWER LIMIT WAS LESS THAN
C    THE LOWER CONFIDENCE LIMIT CALCULATED.
C  ***********
C  VARIABLES:
C    INTEGER:
C      REPS=  AS ABOVE
C      TNFAIL=  AS ABOVE
C      NLL50=NUMBER OF TIMES THE 50 PERCENT LOWER LIMIT
C        CAPTURES THE RELIABILITY.
C      NLL60,NLL70,NLL80,NLL90,NLL95,NLL99= AS NLL50
C        FOR THE HIGHER LIMITS
C    REAL:
C      C= DISPLACEMENT PARAMETER
C      K= SHAPE PARAMETER
C      THETA= SCALE PARAMETER
C      TREL= TRUE RELIABILITY OF THE SYSTEM
C      TFAIL1 TO TFAIL6= TRUE FAILURE VECTORS FOR THE
C        COMPONENTS
C      FC(I),FK(I),FTHETA(I)= FIRST ESTIMATED PARAMETERS
C       OF COMPONENT I
C      SC(I),SK(I),STHETA(I)= AS ABOVE BUT SECOND ESTIMATE
C      SFAIL1 TO SFAIL6= SIMULATED COMPONENT FAILURES
C      RANK(I)=MEDIAN RANK
C      RU= VECTOR OF UNIFORM RANDOM NUMBERS
C      SR1 TO SR6= SIMULATED RELIABILITY OF EACH COMPONENT
C      SRS=SIMULATED RELIABILITY OF SYSTEM
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```
C       FIRST= EXTRAPOLATED VALUE CORRESPONDING TO THE
C         FIRST MEDIAN RANK OF 0
C       LAST= AS ABOVE BUT LAST MEDIAN RANK OF 1
C       SRC=ORDERED RELIABILITY OF EACH COMPONENT WITH
C         EXTRAPOLATED END VALUES
C       SRSORD=AS SRC BUT FOR SYSTEM
C       CL50.....CL99= LOWER CONFIDENCE LIMITS
C*************************************************************
C
      INTEGER  REPS,TNFAIL,NLL50,NLL60,NLL70,NLL80
      INTEGER NLL90,NLL95,NLL99
      INTEGER RGE1,NRUNS
      REAL  R(6),C(6),K(6),THETA(6),TREL, FC(6),FK(6)
      REAL FTHETA(6)
      REAL TFAIL1(100),TFAIL2(100),TFAIL3(100),TFAIL4(100),TFAIL5(100)
      REAL TFAIL6(100),SFAIL1(100),SFAIL2(100),SFAIL3(100),SFAIL4(100)
      REAL SFAIL5(100),SFAIL6(100),SC(6),SK(6),STHETA(6),RANK(602)
      REAL SRSORD(602), RU(6)
      REAL FIRST,LAST,SRS(600),SR1(300),SR2(300)
      REAL SR3(300),SR4(300),SR5(300),SR6(300),SRC(6,302)
      REAL CL50,CL60,CL70,CL80,CL90
      REAL CL95,CL99
      DOUBLE PRECISION DSEED
C  DECLARATIONS COMPLETE
C************************
C
C  INITIALIZE INPUTS AND COUNTERS FOR NUMBER OF TIMES
C  LOWER LIMIT EXCEEDED
      DSEED=17943.0D0
      PRINT*,'L.L.S.   DSEED =  ',DSEED
      RGE1=0
      REPS=150
      NRUNS=150
      PRINT*,'     NRUNS =   ',NRUNS
      TNFAIL=10
      C(1)=10.
      C(2)=0.
      C(3)=15.
      C(4)=30.
      C(5)=25.
      C(6)=50.
      K(1)=2.3
      K(2)=.8
      K(3)=2.0
      K(4)=3.5
      K(5)=1.2
      K(6)=2.
      THETA(1)=200.
      THETA(2)=470.
      THETA(3)=180.
      THETA(4)=120.
      THETA(5)=250.
      THETA(6)=150.
```

```
      NLL50=0
      NLL60=0
      NLL70=0
      NLL80=0
      NLL90=0
      NLL95=0
      NLL99=0
C  CALCULATE THE POINT ESTIMATE OF TRUE SYSTEM RELIABILITY
C
      CALL RLBTY(C,K,THETA,TREL,RGE1)
C GENERATE THE REQUIRED NUMBER OF MONTE CARLO
C  REPETITIONS. FOR EACH RUN DETERMINE IF THE TRUE
C  RELIABILITY IS ABOVE THE LOWER LIMIT AT EACH
C  CONFIDENCE LEVEL AND INCREMENT THE APPROPRIATE
C  COUNTER IF SO.
      DO 5 M=1,NRUNS
C
C  GENERATE THE TRUE COMPONENT FAILURE DATA FOR EACH
C  COMPONENT
      CALL WEIBL(DSEED,C(1),K(1),THETA(1),TNFAIL,TFAIL1)
      CALL WEIBL(DSEED,C(2),K(2),THETA(2),TNFAIL,TFAIL2)
      CALL WEIBL(DSEED,C(3),K(3),THETA(3),TNFAIL,TFAIL3)
      CALL WEIBL(DSEED,C(4),K(4),THETA(4),TNFAIL,TFAIL4)
      CALL WEIBL(DSEED,C(5),K(5),THETA(5),TNFAIL,TFAIL5)
      CALL WEIBL(DSEED,C(6),K(6),THETA(6),TNFAIL,TFAIL6)
C  SORT THE TRUE FAILURE DATA AND CALCULATE THE
C  ESTIMATORS OF THE PARAMETERS
C
      CALL VSRTA(TFAIL1,TNFAIL)
      CALL VSRTA(TFAIL2,TNFAIL)
      CALL VSRTA(TFAIL3,TNFAIL)
      CALL VSRTA(TFAIL4,TNFAIL)
      CALL VSRTA(TFAIL5,TNFAIL)
      CALL VSRTA(TFAIL6,TNFAIL)
      CALL PARAM(TNFAIL,TFAIL1,FC(1),FK(1),FTHETA(1))
      CALL PARAM(TNFAIL,TFAIL2,FC(2),FK(2),FTHETA(2))
      CALL PARAM(TNFAIL,TFAIL3,FC(3),FK(3),FTHETA(3))
      CALL PARAM(TNFAIL,TFAIL4,FC(4),FK(4),FTHETA(4))
      CALL PARAM(TNFAIL,TFAIL5,FC(5),FK(5),FTHETA(5))
      CALL PARAM(TNFAIL,TFAIL6,FC(6),FK(6),FTHETA(6))
C  FOR THE EMPIRICAL DISTRIBUTION, GENERATE REPS
C  RELIABILITY ESTIMATES.
C  FIRST GENERATE TNFAIL SIMULATED FAILURES FOR EACH
C  OF THE COMPONENTS USING THE
C  ESTIMATES OF THE PARAMETERS
      DO 10 L=1,REPS
      CALL WEIBL(DSEED,FC(1),FK(1),FTHETA(1),TNFAIL,SFAIL1)
      CALL WEIBL(DSEED,FC(2),FK(2),FTHETA(2),TNFAIL,SFAIL2)
      CALL WEIBL(DSEED,FC(3),FK(3),FTHETA(3),TNFAIL,SFAIL3)
      CALL WEIBL(DSEED,FC(4),FK(4),FTHETA(4),TNFAIL,SFAIL4)
      CALL WEIBL(DSEED,FC(5),FK(5),FTHETA(5),TNFAIL,SFAIL5)
      CALL WEIBL(DSEED,FC(6),FK(6),FTHETA(6),TNFAIL,SFAIL6)
C  ORDER THE SIMULATED FAILURES AND USE THEM TO OBTAIN
C  THE SECOND ESTIMATE OF THE COMPONENT PARAMETERS
```

```
      CALL VSRTA(SFAIL1,TNFAIL)
      CALL VSRTA(SFAIL2,TNFAIL)
      CALL VSRTA(SFAIL3,TNFAIL)
      CALL VSRTA(SFAIL4,TNFAIL)
      CALL VSRTA(SFAIL5,TNFAIL)
      CALL VSRTA(SFAIL6,TNFAIL)
      CALL PARAM(TNFAIL,SFAIL1,SC(1),SK(1),STHETA(1))
      CALL PARAM(TNFAIL,SFAIL2,SC(2),SK(2),STHETA(2))
      CALL PARAM(TNFAIL,SFAIL3,SC(3),SK(3),STHETA(3))
      CALL PARAM(TNFAIL,SFAIL4,SC(4),SK(4),STHETA(4))
      CALL PARAM(TNFAIL,SFAIL5,SC(5),SK(5),STHETA(5))
      CALL PARAM(TNFAIL,SFAIL5,SC(6),SK(6),STHETA(6))
C  CALCULATE THE RELIABILITY OF EACH COMPONENT
C  USING THE SECOND ESTIMATE OF COMPONENT PARAMETERS
C  TO BUILD THE COMPONENT RELIABILITY VECTORS SR1 TO SR5
C
      SR1(L)=RELY(SC(1),SK(1),STHETA(1),RGE1)
      SR2(L)=RELY(SC(2),SK(2),STHETA(2),RGE1)
      SR3(L)=RELY(SC(3),SK(3),STHETA(3),RGE1)
      SR4(L)=RELY(SC(4),SK(4),STHETA(4),RGE1)
      SR5(L)=RELY(SC(5),SK(5),STHETA(5),RGE1)
      SR6(L)=RELY(SC(6),SK(6),STHETA(6),RGE1)
10    CONTINUE
C
C  ESTABLISH RELIABILITY DISTRIBUTION FUNCTIONS FOR
C  EACH COMPONENT TYPE USING ORDERED MEDIAN RANKS:
C  ONE RANK VECTOR AND THE SIX RELIABILITY DISTRIBUTION
C  VECTORS WILL MAKE UP THE SIX DISTRIBUTION FUNCTIONS
C  EACH WITH REPS+2 VALUES
C
      RANK(1)=0.
      RANK(REPS+2)=1.
      DO 15 I=1,REPS
      RANK(I+1)=(REAL(I)-.3)/(REAL(REPS)+.4)
15    CONTINUE
C  ORDER THE RELIABILITY VECTORS AND ESTABLISH THE VALUES
C  CORRESPONDING TO MEDIAN RANKS 0 AND 1
C
      CALL VSRTA(SR1,REPS)
      CALL VSRTA(SR2,REPS)
      CALL VSRTA(SR3,REPS)
      CALL VSRTA(SR4,REPS)
      CALL VSRTA(SR5,REPS)
      CALL VSRTA(SR6,REPS)
      CALL EXTRA(SR1,RANK,FIRST,LAST,REPS)
      SRC(1,1)=FIRST
      SRC(1,REPS+2)=LAST
      CALL EXTRA(SR2,RANK,FIRST,LAST,REPS)
      SRC(2,1)=FIRST
      SRC(2,REPS+2)=LAST
      CALL EXTRA(SR3,RANK,FIRST,LAST,REPS)
      SRC(3,1)=FIRST
      SRC(3,REPS+2)=LAST
      CALL EXTRA(SR4,RANK,FIRST,LAST,REPS)
      SRC(4,1)=FIRST
      SRC(4,REPS+2)=LAST
      CALL EXTRA(SR5,RANK,FIRST,LAST,REPS)
      SRC(5,1)=FIRST
      SRC(5,REPS+2)=LAST
```

```
      CALL EXTRA(SR6,RANK,FIRST,LAST,REPS)
      SRC(6,1)=FIRST
      SRC(6,REPS+2)=LAST
      DO 20 I=1,REPS
      SRC(1,I+1)=SR1(I)
      SRC(2,I+1)=SR2(I)
      SRC(3,I+1)=SR3(I)
      SRC(4,I+1)=SR4(I)
      SRC(5,I+1)=SR5(I)
      SRC(6,I+1)=SR6(I)
20    CONTINUE
C  RANDOMLY SELECT A RELIABILITY FROM EACH RELIABILITY
C  DISTRIBUTION FUNCTION USING A UNIFORM(0,1) GENERATOR
C  TO OBTAIN A MEDIAN RANK, AND LINEAR INTERPOLATION.
C  CALCULATE SIMULATED SYSTEM RELIABILITY SRS. REPEAT
C  600 TIMES.
C
      DO 25 I=1,600
      CALL GGUBS(DSEED,6,RU)
      DO 30 J=1,6
      II=2
35    IF(II.LE.REPS+2)THEN
         IF(RU(J).LE.RANK(II))THEN
         CALL INTERP(RANK(II),RANK(II-1),SRC(J,II),SRC(J,II-1),
     1RU(J),R(J))
         II=REPS+3
         ELSE
         II=II+1
         END IF
      GO TO 35
      END IF
30    CONTINUE
      SRS(I)=R(1)*(((R(2)+R(3)-R(2)*R(3))*R(4))+(R(5)+R(6)
     1-R(5)*R(6))-((R(2)+R(3)-R(2)*R(3))*R(4))*
     1(R(5)+R(6)-R(5)*R(6)))
25    CONTINUE
C
C  ORDER THE SYSTEM (600) RELIABILITY ESTIMATES USING
C  MEDIAN RANKS AND DETERMINE THE 99,95,90,80,70,60,50
C  PERCENT LOWER LIMITS. NOTE IF EACH CONTAINS THE TRUE
C  SYSTEM RELIABILITY AND IF SO, INCREMENT THE APPROPRIATE
C  COUNTER NLL50,...NLL99.
      CALL VSRTA(SRS,600)
      RANK(1)=0.
      RANK(602)=1.
      DO 40 I=1,600
      RANK(I+1)=(REAL(I)-.3)/600.4
40    CONTINUE
      CALL EXTRA(SRS,RANK,FIRST,LAST,600)
      SRSORD(1)=FIRST
      SRSORD(602)=LAST
      DO 45 I=1,600
      SRSORD(I+1)=SRS(I)
45    CONTINUE
```

```
      CALL INTERP(RANK(8),RANK(7),SRSORD(8),SRSORD(7),
     1.01,CL99)
      CALL INTERP(RANK(32),RANK(31),SRSORD(32),SRSORD(31),
     1.05,CL95)
      CALL INTERP(RANK(62),RANK(61),SRSORD(62),SRSORD(61),
     1.1,CL90)
      CALL INTERP(RANK(122),RANK(121),SRSORD(122),SRSORD(121),
     1.2,CL80)
      CALL INTERP(RANK(182),RANK(181),SRSORD(182),SRSORD(181),
     1.3,CL70)
      CALL INTERP(RANK(242),RANK(241),SRSORD(242),SRSORD(241),
     1.4,CL60)
      CALL INTERP(RANK(302),RANK(301),SRSORD(302),SRSORD(301),
     1.5,CL50)
      IF(CL99.LT.TREL)NLL99=NLL99+1
      IF(CL95.LT.TREL)NLL95=NLL95+1
      IF(CL90.LT.TREL)NLL90=NLL90+1
      IF(CL80.LT.TREL)NLL80=NLL80+1
      IF(CL70.LT.TREL)NLL70=NLL70+1
      IF(CL60.LT.TREL)NLL60=NLL60+1
      IF(CL50.LT.TREL)NLL50=NLL50+1
5     CONTINUE
C  PRINT THE NUMBER OF TIMES THE TRUE RELIABILITY
C  WAS OVER EACH CONFIDENCE LIMIT
      PRINT*,'  TRUE RELIABILITY IS  ',TREL
      PRINT*,'NUMBER OF FAILURES  ',TNFAIL
      PRINT*,'  NUMBER OF REPETITIONS  ',REPS
      PRINT*,'NUMBER OF TIMES RELIABILITY GE 1 =  ',RGE1
      PRINT*,'  NUMBER ABOVE 50 PERCENT LOWER LIMIT  ',NLL50
      PRINT*,'  NUMBER ABOVE 60 PERCENT LOWER LIMIT  ',NLL60
      PRINT*,'  NUMBER ABOVE 70 PERCENT LOWER LIMIT  ',NLL70
      PRINT*,'  NUMBER ABOVE 80 PERCENT LOWER LIMIT  ',NLL80
      PRINT*,'  NUMBER ABOVE 90 PERCENT LOWER LIMIT  ',NLL90
      PRINT*,'  NUMBER ABOVE 95 PERCENT LOWER LIMIT  ',NLL95
      PRINT*,'  NUMBER ABOVE 99 PERCENT LOWER LIMIT  ',NLL99
      STOP
      END
C***************
      SUBROUTINE EXTRA(X,Y,FIRST,LAST,REPS)
C  USES LINEAR INTERPOLATION OFF THE TWO END
C  VALUES TO OBTAIN THE RELIABILITIES CORRESPONDING
C  TO THE MEDIAN RANKS 0 AND 1, FIRST AND LAST.
C  SLOPE IS THE SLOPE OF THE LINEAR LINE BETWEEN THE
C  TWO END VALUES AT EACH END.
      INTEGER REPS
      REAL X(300),Y(302),FIRST,LAST
      Z=X(2)-X(1)
      ZZ=X(REPS)-X(REPS-1)
      V=Y(3)-Y(2)
      VV=Y(REPS+1)-Y(REPS)
      IF(Z.GT.0.)THEN
         SLOPE=V/Z
         IF(SLOPE.EQ.0.)THEN
            FIRST=X(1)
```

```
            ELSE
               FIRST=X(1)-Y(2)/SLOPE
            END IF
         ELSE
            FIRST=X(1)
         END IF
         IF(ZZ.GT.0.)THEN
            SLOPE=VV/ZZ
            IF(SLOPE.EQ.0.)THEN
               LAST=X(REPS)
            ELSE
               LAST=X(REPS)+((1-Y(REPS+1))/SLOPE)
            END IF
         ELSE
            LAST=X(REPS)
         END IF
         IF(FIRST.LT.0.)FIRST=0.
         IF(LAST.GT.1.)LAST=1.
         RETURN
         END
C*****************************************
      SUBROUTINE INTERP(UPPERY,LOWERY,UPPERR,LOWERR,MED,R)
C  USES LINEAR INTERPOLATION TO OBTAIN A RELIABILITY
C  R, CORRESPONDING TO THE MEDIAN RANK, MED. MED IS
C  BRACKETED BY MEDIAN RANKS UPPERY AND LOWERY WHICH
C  CORRESPOND TO RELIABILITIES UPPERR AND LOWERR.
      REAL UPPERY,LOWERY,UPPERR,LOWERR,R,SLOPE,MED
      X=UPPERR-LOWERR
      Y=UPPERY-LOWERY
      IF(X.GT.0.)THEN
         SLOPE=Y/X
         R=LOWERR+((MED-LOWERY)/SLOPE)
      ELSE
         R=UPPERR
      END IF
      IF(R.GT.1.)R=UPPERR
      RETURN
      END
C*********************************
      SUBROUTINE WEIBL(DSEED,C,K,THETA,NW,RW)
C  CALCULATES RANDOM 3-PARAMETER WEIBULL VARIATES
C  RW. A TOTAL OF NW ARE FOUND USING IMSL ROUTINE
C  GGWIB
C  C IS THE POSITION PARAMETER, K IS THE SHAPE, AND
C  THETA IS THE SCALE.
      INTEGER NW
      REAL RW(300),C,K,THETA
      DOUBLE PRECISION DSEED
      CALL GGWIB(DSEED,K,NW,RW)
      DO 3 I=1,NW
      RW(I)=THETA*RW(I)+C
3     CONTINUE
      RETURN
      END
```

```
C*******************
      SUBROUTINE RLBTY(C,K,THETA,RS,RGE1)
C  CALCULATES SYSTEM RELIABILITY FROM COMPONENT
C  PARAMETER DATA.
C  R(6) IS THE COMPONENT RELIABILITIES; RS IS THE SYSTEM.
      REAL C(6),K(6),THETA(6),R(6),RS
      INTEGER RGE1
C  CALCULATE COMPONENT RELIABILITIES
      DO 10 I=1,6
      R(I)=RELY(C(I),K(I),THETA(I),RGE1)
10    CONTINUE
C  CALCULATE SYSTEM RELIABILITY
      RS=R(1)*(((R(2)+R(3)-R(2)*R(3))*R(4))+(R(5)+R(6)
     1-R(5)*R(6))-((R(2)+R(3)-R(2)*R(3))*R(4))*
     1(R(5)+R(6)-R(5)*R(6)))
      RETURN
      END
C*******************
      FUNCTION RELY(C,K,THETA,RGE1)
C  CALCULATES COMPONENT RELIABILITY FOR 3-PARAMETER WEIBULL
      REAL C,K,THETA,T
      INTEGER RGE1
      T=100.
      X=T-C
      IF(X.LE.0.)THEN
         RELY=1.
         RGE1=RGE1+1
      ELSE
         X=((T-C)/THETA)**K
         IF(X.LT.20.)THEN
            RELY=EXP(-X)
         ELSE
            RELY=0.
         END IF
      END IF
      RETURN
      END
```

```
C*************************
      SUBROUTINE PARAM(NUM,FAIL,CW,KW,THETAW)
C MAXIMUM SAMPLE SIZE DIMENSIONED=100
C  INPUT
C  NUM=SAMPLE SIZE
C  FAIL=VECTOR OF SAMPLE DATA
C  OUTPUTS
C  ESTIMATES OF LOCATION PARAMETER CW, SHAPE PARAMETER
C  KW, AND SCALE PARAMETER THETAW
C  VARIABLES:
C   INTEGER
C  NUM=SAMPLE SIZE
C  MIN=MINIMUM SQUARE ERROR
C  FLAG IS A FLAG TO MARK THE MINIMUM ERROR
C   REAL
C  FAIL=VECTOR OF SAMPLE DATA
C  RANK=VECTOR OF MEDIAN RANKS
C  PLOTRK=PLOTTING RANK = LN(LN(1-1/F(T)))
C  EY= VECTOR OF EXPECTED Y VALUES IF THE SAMPLES PLOT
C  LINEARLY ON A WEIBULL PLOT WITH THE Y AXIS BEING
C  THE LN OF THE FAILURE TIMES AND THE X AXIS THE
C  LNLN OF 1/(1-F(T))
C  SQRERR=VECTOR OF SQUARE ERROR SUMS
C  LNFAIL=VECTOR OF LN OF SAMPLE DATA
C  CW,KW,THETAW= AS ABOVE
C  EC,EK,ETHETA= ESTIMATES OF PARAMETERS
C  TOTALX= THE TOTAL OF THE ABSCISSA VALUES
C  TOTALY= THE TOTAL OF THE ORDINATE VALUES
C*****************************************
C
      INTEGER NUM, FLAG
      REAL FAIL(100),RANK(100),PLOTRK(100),EY(0:10)
      REAL SQRERR(0:10),LNFAIL(100),CW,KW,THETAW,MIN
      REAL EC(0:10),EK(0:10),ETHETA(0:10),TOTALX,TOTALY
C  DECLARATIONS COMPLETE
C*************************
C  CREATE A MEDIAN RANK VECTOR AND A PLOTTING LNLN
C  VECTOR WITH A TOTAL FOR THE X AXIS
      TOTALX=0.
      DO 10 I=1,NUM
      RANK(I)=(I-.3)/(NUM+.4)
      PLOTRK(I)=ALOG(ALOG(1./(1.-RANK(I))))
      TOTALX=TOTALX+PLOTRK(I)
10    CONTINUE
C
C  SET VALUES OF C FROM 0 TO 1.*FAIL(1) AND
C  DETERMINE THE SQUARE ERROR FROM A LINEAR
C  PLOT ON WEIBULL PAPER FOR EACH
C  USING THE RELATIONSHIP LN(T)=(1/K)*LNLN(1/1-F(T))
C  +LN(THETA)
```

```
      Z=PLOTRK(NUM)-PLOTRK(1)
      DO 15 J=0,10
      N=1
      IF(J.EQ.10)THEN
         N=2
         Z=PLOTRK(NUM)-PLOTRK(2)
      END IF
      EC(J)=FAIL(1)*.1*REAL(J)
      DO 20 I=N,NUM
      LNFAIL(I)=ALOG(FAIL(I)-EC(J))
20    CONTINUE
C  FIND THE RELATIONSHIP THAT IS CLOSEST TO LINEAR
C  AND USE THE PARAMETERS FOR THAT MATCH
C  FIRST ESTIMATE K, THEN THETA ASSUMING THAT
C  1/K IS THE SLOPE OF THE LINE
      EK(J)=Z/(LNFAIL(NUM)-LNFAIL(N))
      TOTALY=0.
      DO 25 I=N,NUM
      TOTALY=TOTALY+LNFAIL(I)
25    CONTINUE
      ETHETA(J)=(TOTALY-TOTALX/EK(J))/REAL(NUM-N+1)
C  CALCULATE THE ESTIMATES OF LN(T-C) WHICH IS THE ORDINATE.
C  ESTIMATE THE ERROR SQUARED
      SQRERR(J)=0.
      DO 30 I=N,NUM
      EY(I)=PLOTRK(I)/EK(J)+ETHETA(J)
      SQRERR(J)=SQRERR(J)+(EY(I)-LNFAIL(I))*(EY(I)-LNFAIL(I))
30    CONTINUE
15    CONTINUE
C  FIND THE CLOSEST LINEAR RELATIONSHIP AND USE THE
C  PARAMETERS AS THE ESTIMATES
      FLAG=0
      MIN=SQRERR(0)
      DO 35 I=1,10
      IF(SQRERR(I).LT.MIN)THEN
         MIN=SQRERR(I)
         FLAG=I
      END IF
35    CONTINUE
      CW=EC(FLAG)
      KW=EK(FLAG)
      THETAW=EXP(ETHETA(FLAG))
      RETURN
      END
```

```
C*************************
      SUBROUTINE PARAM(NUM,FAIL,CW,KW,THETAW)
C MAXIMUM SAMPLE SIZE DIMENSIONED=100
C  INPUT
C  NUM=SAMPLE SIZE
C  FAIL=VECTOR OF SAMPLE DATA
C  OUTPUTS
C  ESTIMATES OF LOCATION PARAMETER CW, SHAPE PARAMETER
C  KW, AND SCALE PARAMETER THETAW
C  VARIABLES:
C   INTEGER
C  NUM=SAMPLE SIZE
C   REAL
C  FAIL=VECTOR OF SAMPLE DATA
C  RANK=VECTOR OF MEDIAN RANKS
C  PLOTRK=PLOTTING RANK = LN(LN(1-1/F(T)))
C  LNFAIL=VECTOR OF LN OF SAMPLE DATA
C  CW,KW,THETAW= AS ABOVE
C  TOTALX= THE TOTAL OF THE ABSCISSA VALUES
C  TOTALY= THE TOTAL OF THE ORDINATE VALUES
C*****************************************
C
      INTEGER NUM
      REAL FAIL(100),RANK(100),PLOTRK(100)
      REAL SLOPE,LNFAIL(100),CW,KW,THETAW,TOTALX,TOTALY
C  DECLARATIONS COMPLETE
C*****************************
C  CREATE A MEDIAN RANK VECTOR AND A PLOTTING LNLN
C  VECTOR WITH A TOTAL FOR THE X AXIS
      TOTALX=0.
      DO 10 I=1,NUM
      RANK(I)=(I-.3)/(NUM+.4)
      PLOTRK(I)=ALOG(ALOG(1./(1.-RANK(I))))
      TOTALX=TOTALX+PLOTRK(I)
10    CONTINUE
C  ESTIMATE C BY LINEAR EXTRAPOLATION FROM THE FIRST
C  TWO ORDER STATISTICS
      CW=FAIL(2)-(FAIL(2)-FAIL(1))*RANK(2)/(RANK(2)-RANK(1))
C  CALCULATE THE LOG OF THE FAILURE TIMES MINUS C
C  AND THE TOTAL FOR THE Y AXIS
      TOTALY=0.
      DO 25 I=1,NUM
      LNFAIL(I)=ALOG(FAIL(I)-CW)
      TOTALY=TOTALY+LNFAIL(I)
25    CONTINUE
```

```
C CALCULATE K BY THE AVERAGE SLOPE INVERTED
C DETERMINE THE SUM OF ALL THE SLOPES AND THE AVERAGE
C INVERTED
      SLOPE=0.
      DO 20 M=2,NUM
      SLOPE=SLOPE+(LNFAIL(M)-LNFAIL(M-1))/(PLOTRK(M)-PLOTRK(M-1))
20    CONTINUE
      KW=(REAL(NUM)-1.)/SLOPE
C LN(THETA) IS A CONSTANT WHICH IS INCLUDED IN THE
C THE RELATIONSHIP LN(T)=1/K*LNLN(1/(1-F(T)))
C +LN(THETA). IN THE CALCULATIONS OF THE TOTALS
C IT IS INCLUDED NUM TIMES, SO THE AVERAGE VALUE
C WILL EQUAL LN(THETA)
      THETAW=(TOTALY-TOTALX/KW)/REAL(NUM)
      THETAW=EXP(THETAW)
      RETURN
      END
```

```
C*****************************
      SUBROUTINE PARAM(NUM,FAIL,CW,KW,THETAW)
C MAXIMUM SAMPLE SIZE DIMENSIONED=100
C  INPUT
C  NUM=SAMPLE SIZE
C  FAIL=VECTOR OF SAMPLE DATA
C  OUTPUTS
C  ESTIMATES OF LOCATION PARAMETER CW, SHAPE PARAMETER
C  KW, AND SCALE PARAMETER THETAW
C  VARIABLES:
C   INTEGER
C  NUM=SAMPLE SIZE
C   REAL
C  FAIL=VECTOR OF SAMPLE DATA
C  RANK=VECTOR OF MEDIAN RANKS
C  FAILMC= FAILURE TIMES MINUS C
C  LNFAIL=VECTOR OF LN OF SAMPLE DATA
C  CW,KW,THETAW= AS ABOVE
C  SUM1=NUM*SUM OF FAILURE TIMES**K *LN(FAILURE TIMES)
C  SUM2=SUM OF FAILURE TIMES**K
C  TOTALX=SUM OF LN(FAILURE TIMES)
C  SUM0=SUM1/SUM2 -TOTALX
C*****************************************
C
      INTEGER NUM
      REAL FAILMC(100),SUM1,SUM2,SUM0
      REAL FAIL(100),RANK(2)
      REAL LNFAIL(100),CW,KW,EK,THETAW,TOTALX,X
C  DECLARATIONS COMPLETE
C*****************************
C
      DO 10 I=1,2
      RANK(I)=(I-.3)/(NUM+.4)
10    CONTINUE
C  ESTIMATE C BY LINEAR EXTRAPOLATION FROM THE FIRST
C  TWO ORDER STATISTICS
      CW=FAIL(2)-(FAIL(2)-FAIL(1))*RANK(2)/(RANK(2)-RANK(1))
C  CALCULATE THE LOG OF THE FAILURE TIMES MINUS C
C  AND THE TOTAL LOG OF THE FAILURES MINUS C
      TOTALX=0.
      DO 15 I=1,NUM
      FAILMC(I)=FAIL(I)-CW
      LNFAIL(I)=ALOG(FAILMC(I))
      TOTALX=TOTALX+LNFAIL(I)
15    CONTINUE
```

```
C  CALCULATE K BY AN ITERATIVE TECHNIQUE OF MAXIMUM
C  LIKELIHOOD.
       KW=1.
       EK=2.
90     IF(ABS(KW-EK).LT..00001)GO TO 80
       SUM1=0.
       SUM2=0.
       DO 20 I=1,NUM
       X=FAILMC(I)**KW
       SUM1=SUM1+LNFAIL(I)*X
       SUM2=SUM2+X
20     CONTINUE
       SUM0=((SUM1*REAL(NUM))/SUM2)-TOTALX
       EK=REAL(NUM)/SUM0
       KW=(2.*EK+KW)/3.
       GO TO 90
C CALCULATE THETA USING THE VALUE OF K
C AND THE SUMS DETERMINED
80     THETAW=(SUM2/REAL(NUM))**(1./KW)
       RETURN
       END
```

```
      PROGRAM PARAM
C  INPUT
C  N=SAMPLE SIZE (BEFORE CENSORING), N=100 OR LESS AS
C       DIMENSIONED
C  SS1=0 IF SCALE PARAMETER THETA IS KNOWN
C  SS1=1 IF THETA IS TO BE ESTIMATED
C  SS2=0 IF SHAPE PARAMETER K IS KNOWN
C  SS2=1 IF K IS TO BE ESTIMATED
C  SS3=0 IF LOCATION PARAMETER C IS KNOWN
C  SS3=1 IF C IS TO BE ESTIMATED
C  T(I)=I'TH ORDER STATISTIC OF SAMPLE (I=1,N)
C  M=NUMBER OF OBSERVATIONS REMAINING AFTER CENSORING
C       N-M FROM ABOVE
C  C(1)=INITIAL ESTIMATE OR KNOWN VALUE OF C
C  THETA(1)=INITIAL ESTIMATE OR KNOWN VALUE OF THETA
C  EK(1)=INITIAL ESTIMATE OR KNOWN VALUE OF K
C  MR=NUMBER OF OBSERVATIONS CENSORED FROM BELOW,
C  NORMALLY 0 INITIALLY
C  **********************************
C  OUTPUT
C  N,SS1,SS2,SS3,M,C(1),THETA(1),EK(1),MR
C       --SAME AS FOR INPUT
C  C(J)=ESTIMATE AFTER J-1 ITERATIONS
C       (OR KNOWN VALUE OF C)
C  THETA(J)=ESTIMATE AFTER J-1 ITERATIONS
C       (OR KNOWN VALUE OF THETA)
C  EK(J)=ESTIMATE AFTER J-1 ITERATIONS
C       (OR KNOWN VALUE OF K)
C  MAXIMUM VALUE OF J DIMENSIONED IS 550
C  EL=NATURAL LOG OF LIKELIHOOD FOR C(J), THETA(J), EK(J)
C  REFERENCE
C  HARTER, H.LEON AND A.H.MOORE. MAXIMUM LIKELIHOOD
C  ESTIMATORS OF THE PARAMETERS OF GAMMA AND WEIBULL
C  POPULATIONS FROM COMPLETE AND FROM CENSORED SAMPLES
C  TECHNOMETRICS, 7(1965)
C*************************************************************
      DIMENSION T(100),C(550),THETA(550),EK(550),X(56),Y(55)
      SS1=1.
      SS2=1.
      SS3=1.
      N=10
      M=10
      THETA(1)=1.
      C(1)=0.
      EK(1)=1.
      MR=0
      T(1)=146.96
      T(2)=162.52
      T(3)=175.64
      T(4)=220.447
      T(5)=223.9
```

```
      T(6)=261.63
      T(7)=329.99
      T(8)=334.266
      T(9)=350.70
      T(10)=359.14
      IF(N)66,66,104
104   EN=N
      IF(M)66,66,110
110   EM=M
31    ELNM=0.
      EMR=MR
      MRP=MR+1
33    NM=N-M+1
      DO 34 I=NM,N
      EI=I
34    ELNM=ELNM+ALOG(EI)
      IF(MR)66,35,74
74    DO 75 I=1,MR
      EI=I
75    ELNM=ELNM-ALOG(EI)
35    DO 30 J=1,550
      IF(J-1)66,25,37
37    JJ=J-1
      SK=0.
      SL=0.
      DO 6 I=MRP,M
6     SK=SK+(T(I)-C(JJ))**EK(JJ)
      IF(SS1)7,7,8
7     THETA(J)=THETA(JJ)
      GO TO 9
8     IF(MR)66,19,20
19    THETA(J)=((SK+(EN-EM)*(T(M)-C(JJ))**EK(JJ))/EM)
     1**(1./EK(JJ))
      GO TO 9
20    X(1)=THETA(JJ)
      LS=0
      DO 21 L=1,55
      LL=L-1
      LP=L+1
      X(LP)=X(L)
      ZRK=((T(MRP)-C(JJ))/X(L))**EK(JJ)
      Y(L)=-EK(JJ)*(EM-EMR)/X(L)+EK(JJ)*SK/X(L)**(EK(JJ)+1.)
     1+E((JJ)*(EN-EM)*(T(M)-C(JJ))**EK(JJ)/X(L)**(EK(JJ)+1.)
     1-EMR*EK(JJ)*ZRK*EXP(-ZRK)/(X(L)*(1.-EXP(-ZRK)))
      IF(Y(L))53,73,54
53    LS=LS-1
      IF(LS+L)58,55,58
54    LS=LS+1
      IF(LS-L)58,56,58
55    X(LP)=.5*X(L)
      GO TO 61
56    X(LP)=1.5*X(L)
```

```
      GO TO 61
58    IF(Y(L)*Y(LL))60,73,59
59    LL=LL-1
      GO TO 58
60    X(LP)=X(L)+Y(L)*(X(L)-X(LL))/(Y(LL)-Y(L))
61    IF(ABS(X(LP)-X(L))-1.E-4)73,73,21
21    CONTINUE
73    THETA(J)=X(LP)
9     EK(J)=EK(JJ)
10    IF(SS2)12,12,11
11    DO 17 I=MRP,M
17    SL=SL+ALOG(T(I)-C(JJ))
      X(1)=EK(J)
      LS=0
      DO 51 L=1,55
      SLK=0.
      DO 18 I=MRP,M
18    SLK=SLK+(ALOG(T(I)-C(JJ))-ALOG(THETA(J)))*(T(I)-C(JJ))
     1**X(L)
      LL=L-1
      LP=L+1
      X(LP)=X(L)
      ZRK=((T(MRP)-C(JJ))/THETA(J))**X(L)
      Y(L)=(EM-EMR)*(1./X(L)-ALOG(THETA(J)))+SL-SLK/THETA(J)
     1**X(L)+(EN-EM)*(ALOG(THETA(J))-ALOG(T(M)-C(JJ)))*(T(M)
     1-C(JJ))**X(L)/THETA(J)**X(L)+EMR*ZRK*(ALOG(ZRK)/X(L))
     1*EXP(-ZRK)/(1.-EXP(-ZRK))
      IF(Y(L))43,52,44
43    LS=LS-1
      IF(LS+L)47,45,47
44    LS=LS+1
      IF(LS-L)47,46,47
45    X(LP)=.5*X(L)
      GO TO 50
46    X(LP)=1.5*X(L)
      GO TO 50
47    IF(Y(L)*Y(LL))49,52,48
48    LL=LL-1
      GO TO 47
49    X(LP)=X(L)+Y(L)*(X(L)-X(LL))/(Y(LL)-Y(L))
50    IF(ABS(X(LP)-X(L))-1.E-4)52,52,51
51    CONTINUE
52    EK(J)=X(LP)
12    C(J)=C(JJ)
62    IF(SS3)25,25,14
14    IF(1.-EK(J))16,78,78
78    IF(SS1+SS2)57,57,16
16    X(1)=C(J)
      LS=0
      DO 23 L=1,55
      SKL=0
      SR=0
```

```
      DO 15 I=MRP,M
      SK1=SK1+(T(I)-X(L))**(EK(J)-1.)
15    SR=SR+1./(T(I)-X(L))
      LL=L-1
      LP=L+1
      X(LP)=X(L)
      ZRK=((T(MRP)-X(L))/THETA(J))**EK(J)
      Y(L)=(1.-EK(J))*SR+EK(J)*(SK1+(EN-EM)*(T(M)-X(L))
     1**(EK(J)-1.))/THETA(J)**EK(J)-EMR*EK(J)*ZRK*EXP(-ZRK)
     1/((T(MRP)-X(L))*(1.-EXP(-ZRK)))
      IF(Y(L))39,24,40
39    LS=LS-1
      IF(LS+L)70,41,70
40    LS=LS+1
      IF(LS-L)70,42,70
41    X(LP)=.5*X(L)
      GO TO 22
42    X(LP)=.5*X(L)+.5*T(1)
      GO TO 22
70    IF(Y(L)*Y(LL))72,24,71
71    LL=LL-1
      GO TO 70
72    X(LP)=X(L)+Y(L)*(X(L)-X(LL))/(Y(LL)-Y(L))
22    IF(ABS(X(LP)-X(L))-1.E-4)24,24,23
23    CONTINUE
24    C(J)=X(LP)
      GO TO 25
57    C(J)=T(1)
25    IF(MR)66,38,69
38    DO 63 I=1,M
      IF(C(J)+1.E-4-T(I))68,67,67
67    MR=MR+1
63    C(1)=T(1)
68    IF(MR)66,69,31
69    SK=0.
      SL=0.
      DO 36 I=MRP,M
      SK=SK+(T(I)-C(J))**EK(J)
36    SL=SL+ALOG(T(I)-C(J))
      ZRK=((T(MRP)-C(J))/THETA(J))**EK(J)
      EL=ELNM+(EM-EMR)*(ALOG(EK(J))-EK(J)*ALOG(THETA(J)))+
     1(EK(J)-1.)*SL-(SK+(EN-EM)*(T(M)-C(J))**EK(J))/(THETA
     1(J)**EK(J))+EMR*ALOG(1.-EXP(-ZRK))
      IF(J-3)30,27,27
27    IF(ABS(C(J)-C(JJ))-1.E-4)28,28,30
28    IF(ABS(THETA(J)-THETA(JJ))-1.E-4)29,29,30
29    IF(ABS(EK(J)-EK(JJ))-1.E-4)66,66,30
30    CONTINUE
66    STOP
      END
```

APPENDIX C Derivation of Maximum Likelihood Equations

The two-parameter Weibull distribution function is

$$f(t;\theta,k) = \frac{kt^{k-1}}{\theta^k} \exp\left[-\left(\frac{t}{\theta}\right)^k\right]$$

Assuming a random sample of n independent failures T_1, T_2, ... T_n, the likelihood function is

$$L(\theta,k) = \prod_{i=1}^{n} \frac{k\, t_i}{\theta^k}^{k-1} \exp\left[-\left(\frac{t_i}{\theta}\right)^k\right]$$

The natural logarithm of the likelihood function can be used to find the maximum likelihood since the maximum point will be the same for both the logarithm and the function. The random samples can be considered constants for the likelihood function. If the partial derivatives of the logarithm of the likelihood function are taken with respect to each of the two parameters and set equal to zero, the two equations can be solved for the two unknown parameters by first solving for θ in terms of k, then substituting and solving for k.

$$\ln L = \sum_{i=1}^{n} \ln\left(\frac{kt_i}{\theta^k}^{k-1} \exp[-\left(\frac{t_i}{\theta}\right)^k]\right)$$

$$= \sum_{i=1}^{n} [\ln k + (k-1) \ln t_i - k \ln \theta - \left(\frac{t_i}{\theta}\right)^k]$$

$$= n(\ln k - k \ln \theta) + \sum_{i=1}^{n} [(k-1) \ln t_i - \left(\frac{t_i}{\theta}\right)^k]$$

$$\frac{\partial \ln L}{\partial \theta} = -\frac{nk}{\theta} + \frac{k}{\theta^{k+1}} \sum_{i=1}^{n} t_i^k \quad \text{set equal to zero}$$

$$0 = -\frac{nk}{\theta} + \frac{k}{\theta^{k+1}} \sum_{i=1}^{n} t_i^k$$

$$n = \frac{1}{\theta^k} \Sigma t_i^k$$

$$\theta = \left[\Sigma \frac{t_i^k}{n}\right]^{1/k}$$

$$\frac{\partial \ln L}{\partial k} = \frac{n}{k} - n \ln \theta + \sum_{i=1}^{n} \left[\ln t_i - \left(\frac{t_i}{\theta}\right)^k \ln\left(\frac{t_i}{\theta}\right)\right]$$

$$= \frac{n}{k} - n \ln \theta + \sum_{i=1}^{n} \ln t_i - \sum_{i=1}^{n} \left(\frac{t_i}{\theta}\right)^k \ln\left(\frac{t_i}{\theta}\right)$$

$$= \frac{n}{k} - \frac{n}{k} \ln \left(\frac{\sum_{i=1}^{n} t_i^k}{n}\right) + \sum_{i=1}^{n} \ln t_i - \frac{n}{\sum_{i=1}^{n} t_i^k} \sum_{i=1}^{n} t_i^k \ln\left(\frac{t_i}{\theta}\right)$$

$$= \frac{n}{k} - \frac{n}{k} \ln\left(\sum_{i=1}^{n} t_i^k\right) + \frac{n}{k} \ln n + \sum_{i=1}^{n} \ln t_i$$

$$- \frac{n}{\sum_{i=1}^{n} t_i^k} \sum_{i=1}^{n} t_i^k \ln t_i + \frac{n}{k \sum_{i=1}^{n} t_i^k} \sum_{i=1}^{n} t_i^k \ln\left(\frac{\sum_{i=1}^{n} t_i^k}{n}\right)$$

$$= \frac{n}{k} + \sum_{i=1}^{n} \ln t_i - \frac{n}{\sum_{i=1}^{n} t_i^k} \sum_{i=1}^{n} t_i^k \ln t_i$$

$$+ \frac{n}{k} \ln n - \frac{n}{k \sum_{i=1}^{n} t_i^k} \sum_{i=1}^{n} t_i^k \ln n + \frac{n}{k \sum_{i=1}^{n} t_i^k} \sum_{i=1}^{n} t_i^k \ln \sum_{i=1}^{n} t_i^k$$

$$- \frac{n}{k} \ln \sum_{i=1}^{n} t_i^k$$

$$= \frac{n}{k} + \sum_{i=1}^{n} \ln t_i - \frac{n}{\sum_{i=1}^{n} t_i^k} \sum_{i=1}^{n} t_i^k \ln t_i$$

set equal to zero and solve

$$-\frac{n}{k} = \sum_{i=1}^{n} \ln t_i - \frac{n}{\sum_{i=1}^{n} t_i^k} \sum_{i=1}^{n} t_i^k \ln t_i$$

$$k = \frac{n}{\dfrac{n \sum_{i=1}^{n} t_i^k \ln t_i}{\sum_{i=1}^{n} t_i^k} - \sum_{i=1}^{n} \ln t_i}$$

APPENDIX D Component Failure Data - Time To Failure

Component 1

139	146	162	175	220	233	261	290
308	329	334	335	350	359	463	

Component 2

113	216	246	252	296	326	415	431
444	480	529	535	661	664	995	

Component 3

165	174	272	289	305	348	353	371
384	462	463	508	510	518	570	

Component 4

92	99	142	150	151	152	160	167
182	187	200	201	211	214	216	

Component 5

160	186	200	204	206	213	259	261
265	287	298	304	326	327	438	

Component 6

107	111	142	162	189	212	231	233
266	274	306	369	498	509	719	

VITA

Murray Ross MacDonald joined the Royal Canadian Air Force in April 1964. He completed navigation training and received his wings in June 1967 after which he served as a navigator and tactical coordinator on Neptune and Argus long-range patrol aircraft. He completed the year-long Aerospace Systems Course in June 1975, then served as a project officer in the Aerospace Engineering Test Establishment. In June 1980 he completed a Bachelor of Science in Mathematics degree. He was employed in NORAD Headquarters as the Branch Head for deep space, then the Division Chief for the Space Detection and Tracking network until entering the School of Engineering, Air Force Institute of Technology, in June 1981.

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A Double Monte Carlo method of obtaining confidence limits for complex systems based on component failure data assuming a three parameter Weibull distribution was developed. Three new parameter estimation routines were developed and compared with the Harter-Moore three parameter maximum likelihood routine for use with the Monte Carlo method. The sensitivity of the method to system reliability, sample size, and number of points in the component reliability distributions was assessed. An approximate method of calculating and correcting for parameter estimation bias was developed and illustrated.

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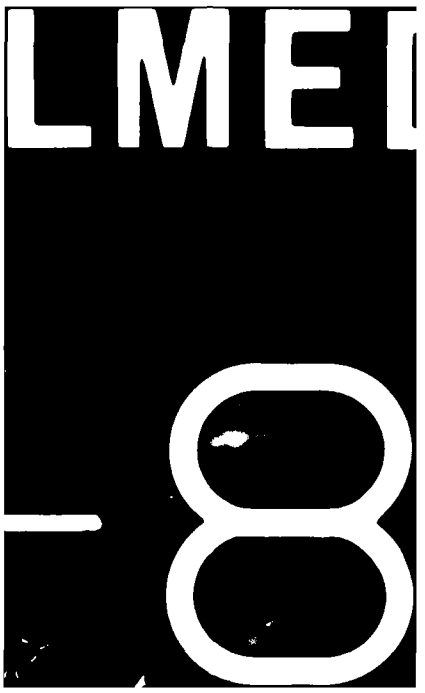
The Double Monte Carlo method appears to be effective at system reliabilities from 74% to 96% with component failure sample sizes as small as five with the Linear Least Squares parameter estimation routine developed.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